

5-2009

# A Characterization of Optimal Strategies in a Reciprocal Product Dumping Environment

Bryan Callaway  
*College of William and Mary*

Follow this and additional works at: <http://publish.wm.edu/honorsthesis>

---

## Recommended Citation

Callaway, Bryan, "A Characterization of Optimal Strategies in a Reciprocal Product Dumping Environment" (2009). *Undergraduate Honors Theses*. Paper 337.  
<http://publish.wm.edu/honorsthesis/337>

This Honors Thesis is brought to you for free and open access by the Theses, Dissertations, & Master Projects at W&M Publish. It has been accepted for inclusion in Undergraduate Honors Theses by an authorized administrator of W&M Publish. For more information, please contact [wmpublish@wm.edu](mailto:wmpublish@wm.edu).

**A Characterization of Optimal Strategies in a Reciprocal Product Dumping Environment**

A thesis submitted in partial fulfillment of the requirement  
for the degree of Bachelors of Arts in Economics from  
The College of William and Mary

by

Bryan White Callaway

Accepted for \_\_\_\_\_  
(Honors)

\_\_\_\_\_  
Arnab Basu, Director

\_\_\_\_\_  
T.J. Cheng

\_\_\_\_\_  
David Feldman

Williamsburg, VA  
April 30, 2009

## TABLE OF CONTENTS

Table of contents .....	II
Acknowledgements .....	III
Chapter I: Introduction .....	IV
Chapter II: The Brander-Krugman Framework, <i>revisited</i> .....	VIII
Section 2.1 .....	VIII
The Basic Model Under Cost Asymmetry	
Section 2.2 .....	X
The Optimal Levels of Output	
Section 2.3 .....	XVI
Concluding Remarks	
Chapter III: The Brander-Krugman Model as a Repeated Game .....	XVII
Section 3.1 .....	XVII
Reviewing Pinto (1986)	
Section 3.2 .....	XIX
The Sequential Game Model	
Section 3.3 .....	XXI
Deriving the Equilibrium Strategies	
Section 3.4 .....	XXVI
The Nash Equilibrium: An Optimal Strategy Profile	
Section 3.5 .....	XXVII
Concluding Remarks	
Conclusion.....	XXIX
References .....	XXXI
Appendix .....	XXXII

## Acknowledgements

I would like to thank my advisor, Arnab Basu, for providing me with invaluable assistance throughout the duration of this project. In addition, I would also like to thank the members of my Honors Committee for their time, patience, and assistance throughout this process.

## I. Introduction

Condemned by the World Trade Organization and governments across the world, ‘dumping’ in international trade has become an ever-expanding subject of debate by economists and policymakers, alike. Widely considered to be an anti-competitive practice, product dumping is a form of monopolistic price discrimination, which arises as a result of discriminable domestic and export markets. Standard equilibrium theory holds that, in the context of international trade, perceived differences in the elasticities of demand in these two markets will enable a producer to price discriminate its product between domestic and international markets. Dumping is a special analog of this, and is defined as an instance in imperfectly competition, where the elasticity of demand for the international market exceeds that of its domestic counterpart, thereby enabling a producer to sell its product more cheaply abroad than it does in its own domestic market. Despite being a natural product of free trade, dumping has been frowned upon by regulators who state that it amounts to nothing more than an anti-competitive trade practice.

Since the first documented cases of product dumping in the late eighteenth-century, which involved British exporters ‘dumping’ goods in the United States, policymakers have decried its existence. Since the first anti-dumping legislation was enacted in Canada in 1904, an innumerable number of international agreements have emerged, such as the General Agreement on Tariffs and Trade (GATT) and the Uruguay Round Anti-Dumping Agreement [6]. The purpose of these agreements has generally been to define a legal definition of what product dumping is, as well as a uniform set of antidumping policies. These antidumping policies, which generally take shape in the form of tariffs and other prohibitive barriers, are frequently the subject of rather contentious debate. A significant reason for this contention can be attributed to the varying cardinalities of ‘dumping’, and the convoluted economic justifications for policing each of them. Regardless of the justifications that have been propounded, economists tend to agree that antidumping legislation is as unnecessarily costly as it is

prohibitive to free trade. Indeed, there are very few circumstances in which anti-dumping legislation is by any means justified [2]. Even then, such situations are as implausible as they are rare. Take for example the case of what is known as “predatory dumping”. Widely considered to be the most egregious form of dumping, predatory dumping is the international analog to domestic predatory pricing, as it is a firm’s intention to deliberately sell its product at sub-competitive prices in an attempt to drive the domestic producers out of the market. Nevertheless, despite being found in fewer than 5% of all alleged dumping charges in the United States, predatory dumping continues to be a central justification for the continued existence of antidumping policies [2].

Indeed, given the rarity of these most egregious dumping practices the economic justifications for antidumping policies become even more untenable. However, proponents of such policies contend that dumping practices result in a loss of profits for the domestic industry of the importing nation [2]. While there are undeniably instances in which this can occur, their existence is difficult to define, and are based almost exclusively off of a “fair price” metric, which seek to estimate a foreign firm’s production costs [2]. Like the antidumping policies that spawned their creation, these “fair price” metrics are themselves rather untenable, as they generally do not account for the nature of the markets in which this price discrimination is occurring. This methodology is fundamentally flawed since it is in fact market dynamics, and not price alone, that determines whether or not dumping, in the economic sense of the word, can occur in a given market. This flaw is not surprising, for it arises at the precise point where the legal and economic definitions of product dumping diverge. While both the legal and economic definitions of dumping reach the same end, an exporter selling its product for a lower price in its export market than in its home market, the economic definition is fundamentally concerned with why such behavior arises in the first place.

Astoundingly high levels of trade liberalization and deregulation in the last thirty years have exposed trade and industrial organization theory to a plethora of new markets for which the possibility of product dumping must now be

considered. As a result, considerable attention has been given to product dumping in international trade literature in recent decades, especially in light of China's growing economic integration, whose manufacturers are often the source of the growing number of dumping complaints and violations in the United States. Prior to China's entry into the WTO, and its export boom in the latter half of this decade- which has seen export growth rates as high as twenty-five percent per annum- dumping allegations made against China were confined to a relatively small set of industries [3]. However, throughout China's export boom of this decade, which has seen its share of the world market more than double in size, the number of documented cases of dumping by Chinese manufacturers has seen a more than proportional increase, in an even wider array of industries [3]. A prime example of these dumping violations takes shape in the form of the Chinese off-road tire industry, whose manufacturers were recently fined, in the form of duties, by the United States Department of Commerce for price distortion. The U.S. Department of Commerce estimates that Chinese tire producers and exporters were selling their products in the U.S. at rates of 10.98 to 210.48 percent less than their fair-market value [6]. In light of this increase in the number of industries in which product dumping has been alleged, both in the U.S. and elsewhere, it is only natural to investigate how product dumping arises under varying market structures.

One of the first forays into extending the generalized case of product dumping was achieved by Brander and Krugman [1], who extend the product dumping literature to consider a symmetric duopolistic environment in which elasticity of demand perceptions give rise to intra-industry trade. According to these authors, such trade will result in a phenomenon known as 'reciprocal dumping' if both firms perceive that the elasticity of demand in their export markets is higher than that which they realize in their home market. This is a rather parsimonious conclusion, which can be generalized to nearly any intra-industry trade scenario. In turn, this begs the question, how can this framework be characterized when we weaken the assumptions made by Brander and Krugman? The objective of this paper is accordingly to characterize several trade

equilibriums in this duopolistic setting while weakening the symmetry assumptions set forth in Brander and Krugman [1]. To achieve this, we will evaluate their proposed model in the context of both a static equilibrium and sequential game setting. In the sequential game setting, we will show how government regulation of intra-industry trade can positively affect the welfare of the nations involved through a form of information coordination.

This paper is organized as follows. Chapter 1 characterizes a reciprocal product dumping environment in the presence of asymmetric cost functions across firms which experience different economies of scale. Chapter 2 extends the reciprocal dumping scenario characterized in Brander and Krugman [1], but in the context of a finite sequential game where the information set held by both firms is asymmetric. Chapter 2 is followed by a brief conclusion and appendix, which contains the pertinent derivations used in Chapters 1 and 2.

## Chapter I

### *The Brander-Krugman Framework, Revisited*

---

#### I. The Basic Model Under Cost Asymmetry

Using Brander and Krugman [1] as a basic framework, we begin by assuming the existence of two countries and two markets, defined as ‘domestic’ and ‘foreign’ respectively. The firms and markets of these two countries are identical, and firms are assumed to be able to export their products to one another’s markets, freely. In this context, trade is said to be ‘free’ when products may be sold abroad, free from any outside intervention or restrictive policy (e.g. tariffs and quotas). Although trade is assumed to be free, firms incur transport costs in exporting their products abroad.

To consider the case of intra-industry dumping, as Brander and Krugman [1] do, we will restrict our scope to consider only the market in each country for one commodity, which is defined as  $Z$ . Let  $Z$  be a homogenous commodity; that is, units of  $Z$  are assumed to be identical regardless of what firm produces them. In each country, we assume that there is only one producer of  $Z$ , which

correspondingly exhibit oligopolistic behavior as a result of our assumption of open economies. The lynchpin of this duopolistic market structure is the Cournot perception held by each firm. That is, both firms believe that the other's level of output is fixed in each market, until proven otherwise. As a result, both of these firms select their profit-maximizing levels of output subject to their implicit assumption that the other firm is holding its output fixed.

Where the Brander and Krugman [1] model relies on the symmetry of the domestic and foreign firms' cost structures, we relax this assumption, considering instead the feasibility of reciprocal dumping in the case of asymmetric cost functions. To establish this asymmetry, we first consider the case of the domestic firm, which we assume to exhibit decreasing economies of scale.

The domestic firm, which produces 'x' units of Z for domestic consumption and 'x\*' units of Z for foreign consumption, exhibits a quadratic total cost function, and thus an increasing marginal cost for both domestic and foreign units. A key implication of the domestic firm's increasing cost-function is that its output functions for each market are implicitly a function of one another.

As explained, both the domestic and foreign firms are exposed to transportation costs associated with their exports. This cost follows directly from Brander and Krugman [1], which can be likened to the 'iceberg' costs first proposed in the Stolper-Samuelson Theorem. Essentially, this cost assumes that some proportion of exports is 'destroyed' during the export process, which we define as:

$$(1) \quad \frac{1}{g}, g \in (0,1]$$

It is important to point out that because  $g$  is bounded between 0 and 1, the transportation cost functions as a multiplier, which is in keeping with the idea of 'iceberg' transport costs. For example, when  $g = 0.9$ , the transport cost of each unit of Z is 1.1, suggesting that 10% of the exported products are 'destroyed' in the process of export

Combining this cost of export with the quadratic cost function for Z, and using  $x^*$  to denote the quantity produced for the foreign market, the domestic firm's total cost function may be represented:

$$(2) \quad c(x + x^*) = (x + x^*)^2 - x^*/g - F$$

Analogous to the domestic firm, the foreign firm produces outputs 'y' and 'y\*' of Z for the domestic and foreign consumption, respectively. The foreign firm is assumed to experience constant returns to scale, and thus a constant marginal cost, c. This is a convenient assumption, which enables the foreign firm to select its profit-maximizing level of output for each market independently of one another. This set of assumptions about the foreign firm's cost function may be expressed mathematically as:

$$(3) \quad c(x + x^*) = (x + x^*) - x^*/g - F$$

It is important to underscore that the solution set of this new model is independent of our cost assumptions for each firm. That is, which firm experiences an increasing cost function (e.g. domestic or foreign) will not affect our analysis. However, unlike Brander and Krugman [1], the asymmetry of this characterization necessitates the uniqueness of the solution sets in each market. As a result, we need to define and solve for the optimal solutions in each of the two markets, as the market shares of each firm will differ for both of the two markets.

## II. The Optimal Levels of Output

In order to identify the trade equilibrium, we must first define the profit functions that are to be maximized. We begin by defining the total revenue functions of both firms. Since we are assuming that both firms are able to trade freely with one another, the total revenue functions incorporate revenue from both

the domestic and foreign markets. To arrive at the profit function of each firm, we simply subtract each firm's total cost function from their total revenue function. Mathematically, we can express this function for the domestic and foreign firm, respectively, as:

$$(4) \quad \pi_H = p_H(x + y)x + p_F(x^* + y^*)x^* - (x + x^*)^2 - x^*/g - F$$

$$(5) \quad \pi_F = p_F(x^* + y^*)y^* + p_H(x + y)y - c(y + y^*/g) - F$$

Using the notation set forth in defining the total cost functions for both the domestic and foreign, asterisks are used to denote variables associated with the foreign country. In addition, we use the variable 'F', to define all fixed costs associated with production. Again, the dependent nature of the domestic firm's quadratic cost structure, along with the cost structure asymmetry of the two firms, reveals the necessity of considering each country's solution set.

The first solution set that we will consider is the simpler case of the domestic market. Derivations for the solution set for the foreign firm are annotated with the letter (A) next to the equation number. With each firm maximizing with respect to their domestic market output, the following first-order conditions are found:

$$(6) \quad \pi'_D = p(x + y)[\varepsilon + \sigma - 1] - 2x\varepsilon - 2x^*\varepsilon = 0$$

$$(6A) \quad \pi'_F = p(x + y)[\varepsilon - \sigma] - (c\varepsilon/g) = 0$$

We similarly compute the first-order conditions in the foreign market:

$$(7) \quad \pi'_D = p^*(x^* + y^*)[\varepsilon - \sigma] - 2x^*\varepsilon - (1/g) = 0$$

$$(7A) \quad \pi'_F = p^*(x^* + y^*)[\varepsilon + \sigma - 1] - c\varepsilon = 0$$

Within both (4) and (5), we use the variable  $\sigma$  to represent the exporting firm's market share in the given market, and similarly use  $\varepsilon$  to denote the point elasticity of demand in the domestic market.

We derive the point elasticity of demand and foreign market share variables from the firms' Total Revenue functions mechanically through the simple inclusion of a multiplicative  $\left(\frac{x+y}{x+y}\right)$  term.<sup>1</sup> The justification for utilizing this term is that it enables us to interpret the marginal revenue function more easily by expressing them in terms of the point elasticity of demand and the exporting firm's market share. This is purely a mechanical nuance, which merely serves to make the trade equilibrium more comprehensible, and has no effect on our marginal revenue solutions.

Because the first-order conditions necessitate the optimization of the total profit function, the equations listed in (4) and (5) may be economically interpreted as best-response functions in their implicit form. It is each firm's adoption of the so-called "Cournot perception" that enables the first-order conditions to function in such a capacity, as each firm assumes that the other is holding their output fixed. Rearranging, and using  $Z$  to denote the total level of output in each market, these best-response functions, we arrive at the following:

$$(8) \quad p(z) = \frac{2(x+x^*)\varepsilon}{\varepsilon + \sigma - 1}$$

$$(8A) \quad p(z) = \frac{c\varepsilon}{g(\varepsilon - \sigma)}$$

$$(9) \quad p(z^*) = \frac{(2x + 2x^* + \frac{1}{g})\varepsilon}{\varepsilon - \sigma}$$

$$(9A) \quad p(z^*) = \frac{c\varepsilon}{(\varepsilon + \sigma - 1)}$$

Rewritten, each of these equations is now an explicit best-response function for the domestic and foreign firms. In this particular instance, they

---

<sup>1</sup> The mathematical derivation of this term is illustrated in the appendix.

denote each firm's profit-maximizing level of output for a given price in each market. An important point to underscore from these solutions is the dependence of (6) and (7) on one another, which prevents us from being able to solve explicitly for the optimal quantities of output for either firm. As a result, though we are able to generalize the trade equilibrium values, we will not be able to develop generalized, explicit solutions for them. Rather, these solutions will depend on the assumptions we make about the demand function in each market. The reason for this is due to the quadratic cost function that we assume for the domestic firm, which prevents it from selecting its domestic quantity independent of its export quantity. Although we are unable to solve for explicit generalized trade equilibrium for all possible demand functions, we are able to do so for linear demand functions. An example of how this is possible is discussed in the Appendix.

While it is unfortunate that we cannot generalize the trade equilibrium across all the set of all possible demand functions, we can still nevertheless derive the implicit trade levels by solving for the size of the exporting firm's market share in each market, which is denoted by  $\sigma$ . Solving for  $\sigma$ , and letting  $\gamma$  equal the domestic firm's total level of output, we arrive at the following trade levels in the domestic market:

$$(10) \quad \sigma_D = \frac{2\gamma g \varepsilon - c\varepsilon + c}{2\gamma g + c}$$

Similarly, we may solve for  $\sigma$  in the foreign market:

$$(11) \quad \sigma_F = \frac{c\varepsilon + 2\gamma + \frac{1}{g} - 2\gamma\varepsilon - \frac{\varepsilon}{g}}{(2\gamma + \frac{1}{g} + c)}$$

When these values are found to be within  $\in (0, 1]$ , and the price in both markets found to be non-negative, we can verify that trade is indeed possible. These resulting values are quite useful since they allow us to determine what properties these two markets must demonstrate in order for intra-industry trade to

exist.<sup>2</sup> More importantly, an analysis of  $\sigma$  enables us to generalize the set of optimal trade levels by studying its levels when tested against the functional limits of the price-elasticity of demand.<sup>3</sup> Since  $\sigma$  only characterizes only part of the full trade equilibrium which we seek to know, we must also solve for the generalized price functions in each of the markets. In the domestic market, the equilibrium price,  $p_D$ , is given as:

$$(12) \quad p_D = \frac{\varepsilon(2\gamma g + c)}{g(2\varepsilon - 1)}$$

And in the foreign market  $p_F$  may be given by:

$$(13) \quad p_F = \frac{\varepsilon(2g\gamma + c)}{g(2\varepsilon - 1)}$$

### *2.1 The Necessary Conditions for Arbitrage*

Owing to the fact that it is only in imperfectly competitive or separable markets that dumping can occur it is only natural to consider the possibility of arbitrage opportunities within them. An arbitrage opportunity arises when there is the existence of an instantaneous risk-free profit. This instantaneous risk-free profit arises from a disparity in the pricing of a commodity across markets. Generally speaking, the instantaneous risk-free profit that emerges as a result of this price disequilibrium exists by purchasing the commodity in the market where it is least expensive, and selling it in the more expensive market. In complete markets, where the number of transactions involved is considerable, such opportunities are few and far between, and any pricing disparities should be erased almost instantaneously owing to the price convergence that results from arbitrage. Establishing the conditions for arbitrage is important for several reasons. However, the most important reason for determining when these

---

<sup>2</sup> By analyzing the roots and limits of both (8) and (9) we are also able to identify circumstances in which two-way trade is not feasible.

<sup>3</sup> These limits are of little interest to us in the context of identifying the trade equilibrium, as they merely illustrate how the trade equilibrium changes as a result of exogenous changes in the market.

opportunities can occur in a market is to establish the set of possible trade equilibrium values. We can do this by assuming that arbitrage opportunities do not exist, and as a result, we can impose restrictions on the set of possible point elasticities of demand at the trade equilibrium.

Using a simple analysis of the pricing disparity in the asymmetric cost model we are considering, we may define the circumstances in which arbitrage opportunities would arise in the market for the commodity,  $z$ . Although we have already established the existence of a pricing disparity between the two markets, the existence of an arbitrage opportunity necessitates that the price spread between these two markets be at least equal to the transportation cost,  $(1/g)$ , as this is the minimum spread that would ensure zero-economic profit:

$$(12) \quad p_{spread} = \frac{c\varepsilon - c\varepsilon g + \varepsilon}{2\varepsilon g - g} \geq \frac{1}{g}$$

The necessary condition for this relation to hold is found to be  $0.5 < \varepsilon \leq 1$ , as it is only when the price elasticity of demand is within this set that an arbitrage profit can exist.<sup>4</sup> It is important to clarify that the aforementioned set is unique to the cost of export, which we assume is the same for all participants in the market. Moreover, if we are to assume free access to this export ability by all market participants, then the Brander-Krugman [1] conclusion that  $\varepsilon > 0.5$  may be viewed as incomplete, if we were to relax the assumption about symmetric markets, to the extent that it does not control for the possibility of arbitrage opportunities, which should theoretically not exist.

## 2.2 *Confirming the Existence of the Equilibrium Points*

To confirm that the values for price and  $\sigma$  function as equilibrium points within their respective markets; it is necessary that we must impose several reasonable restrictions on the higher-order derivatives of the profit functions for

---

<sup>4</sup> The lower-bound of this range is the result of the root in the denominator. As a result, two-way trade can only arise where  $\varepsilon > 0.5$ . The inequality above is also generalized to situations in which  $c \geq 1$ . Otherwise, the inequality is  $0.5 < \varepsilon < \frac{1}{-c+cg+1}$ .

each firm. The first necessary condition that we must consider is the negativity of the second-derivatives of each firm's profit function. Economically, this restriction states that a given profit function is decreasing, and therefore, increases in output would come at the expense of a reduced profit-level. Indeed, this is a very reasonable requirement given the duopoly setting that we are considering. Of equal importance to establishing the stability of the trade equilibrium is the requirement that each of the firm's profit functions, or cross derivatives be negative.<sup>5</sup> Economically this restriction requires that the marginal revenue of a firm declines if the other increases its output. When this condition holds across the set of production levels for both firms, the prescribed trade equilibrium is a unique solution.

#### IV. Concluding Remarks

This chapter characterizes the Brander-Krugman [1] model using a set of significantly weaker assumptions than those that were used in the characterizations of the original 1983 paper. Applying a weaker set of restrictions on this framework is advantageous for several reasons. The most prominent of these is that by using the less restrictive assumption that the firms in the model are not identical, and have different economies of scale, we are able to not only generalize the instances in which economic dumping would ever actually be realized, but also able to characterize the model under a set of more empirically testable assumptions. The constant returns to scale assumption set forth in Brander-Krugman [1] is indeed a very strong assumption.

Finally, our analysis concludes with an examination of the relevant comparative-static derivatives, which are explained in-depth in the Appendix. This analysis is important economically, as it enables us to better understand the rates at which the domestic and foreign markets return to equilibrium in the presence of export subsidies.

---

<sup>5</sup> For the domestic firm, these are defined as:  $\frac{\partial x}{\partial y} < 0$  and  $\frac{\partial x^*}{\partial x^*} < 0$ . For the foreign firm, these derivatives are  $\frac{\partial y}{\partial x} < 0$  and  $\frac{\partial y^*}{\partial x^*} < 0$ .

## Chapter II

### *The Brander-Krugman Model as a Repeated Game*

---

#### I. Reviewing Pinto (1986)

This chapter presents an economic analysis of the original Brander-Krugman [1] model in the context of a finite sequential game, and the threat of government market regulation. With the assistance of several simplifying assumptions that govern the market environment in both the domestic and foreign markets, we will characterize an environment in which the possibility of government regulation will incentivize international trade at the expense of the government's domestic firm. Additionally, we will show that the equilibrium strategies in this environment are in fact optimal.

Despite the parsimony of the Brander and Krugman [1] model, a natural question which arises is, how well does it function in a sequential game environment? Because of the nature of international trade, it is important to

consider the functionality of such a model in the context of non-cooperative game theory. Within this context, each firm, or player, is assumed to have two strategies available to them, diversify by exporting to the market abroad, or assuming an autarky attitude. Fitting this to a non-cooperative game is ideal for reasons stemming from both the oligopolistic nature of the firms in the market, as well as the inherent transportation costs involved with international trade.

One of the first models developed for analyzing reciprocal dumping in this context is found in Pinto [5], which considers reciprocal dumping in the context of an infinite sequential game. Although this work presents several interesting conclusions, including the existence of an autarkic solution as a robust Nash Equilibrium, they are primarily the result of the existence of the transportation costs associated with exports, which follows directly from Brander and Krugman [1]. However, since this cost exists as a multiplier,  $1/g$ , and  $g \in (0, 1]$ , which is a very liberal assumption to say the least, it is indeed very difficult to arrive at any definitive conclusions from the model presented in Pinto [5]. Despite these shortcomings, Pinto [5] nevertheless makes a strong case for considering reciprocal dumping in the context of a sequential game.

Our analysis of reciprocal dumping in this context builds on the original structural model considered by Pinto [5]. However, rather than focus our analysis on the general Cournot model considered by both Brander and Krugman [1] and Pinto [5], we instead choose to consider a sequential Stackelberg game where the government acts as a third party. While a Stackelberg market has previously not been considered within the Brander and Krugman [1] model, it is a worthy assumption since it can be easily applied to markets in which there are new entrants. Presumably, these new entrants are able to ‘learn’ about the other firms currently in the market, and as a result, have an information advantage that is analogous to that of the ‘Follower’ under Stackelberg competition.

Although it was our goal to test the robustness of reciprocal dumping under Stackelberg competition, the introduction of the government as a third party, presents an interesting analysis in and of itself, as we are able to show

characterize a form of government regulation that not only improves the results of intra-industry trade, but also consumer welfare in each of the exporting economies.

## II. The Sequential Game Model

The basic market model we will consider is fundamentally identical to that of Pinto [5], which considers reciprocal dumping in the context of an infinite sequential game. There are two firms and two countries, domestic and foreign. The two firms, which are identical in nature, each of produce a homogenous commodity,  $Z$ . By virtue of their symmetric nature, both firms are dually assumed to exhibit constant returns to scale in production. The markets in each of these two countries are also identical in all aspects. While the two countries are free to trade with one another, there is a per-unit transportation cost,  $\tau$ , associated with doing so. Explicitly, this per-unit transportation cost is defined as:  $\tau = 1/g$ , and functions as a multiplier since  $g \in (0, 1]$ . Combining this transportation cost,  $g$ , with the cost of production, which is assumed to be linear, yields the cost of an exported good, which we define as.

We assume the demand for the commodity  $Z$  to be linear and defined by the inverse function  $p = \alpha - \beta q$  with  $\alpha > 0$ . Since we may simplify our computations without any loss in generalizability, we may extend the without loss of generalizability (WLOG) assumption set forth in Pinto by setting  $\beta = 1$ . By symmetry, these assumptions extend across both markets.

As stated, the cost associated with the production of commodity  $z$  is assumed to be constant. The cost function for the production of  $z$  in autarky is therefore defined as  $c(z) = c(q)$ , which results in constant returns to scale, as marginal cost is a constant,  $c$ .<sup>6</sup> Extending the autarky cost function to the free-

---

<sup>6</sup> We have constant returns to scale (CRS) in this scenario, because we assume that there are no fixed costs.

trade scenario, we arrive at  $c(z) = cq_D + \tau q_F$ , where the marginal cost of an exported unit of  $z$  is a constant,  $\tau$ .

### 2.1 *The Sequential Game Assumptions*

Where Pinto [5] considered an infinite sequential game, we will limit our treatment to a finite horizon with three periods. Additionally, unlike the Cournot duopoly model that Pinto considers, in which the home and exporting firms arrive at the market simultaneously, we assume that each firm moves *first* in its home market. This results with the firms engaging in Stackelberg Competition, as they set their levels of market output sequentially, with the exporting firm selecting its profit-maximizing level of output subject to the quantity that was previously announced by the home firm.

In an additional deviation from the Pinto framework, we assume that the domestic firm perceives the exporting firm's ability to successfully enter the market at each period as  $(P_n = .25(n + 1))$ , where  $n$  is the current period of the game. Thus, the perceived probabilities of successful entry for each of the three periods are, respectively: 0.5, 0.75, and 1. In creating this assumption, we remove Pinto's assumption of perfect information across the two firms, as the actual probability of the exporting firm has the ability to successfully enter the market in any given period,  $(P_n = 1)$ . The final assumption that we consider in our game is the threat of government intervention.

The final assumption we shall impose is that the countries considered in the model are barred from imposing import or export restraints, but are nevertheless interested in dissuading imports of  $Z$ . This assumption is analogous to two countries engaged in a free-trade agreement in which such practices (e.g. tariffs and quotas) are barred. Since the governments of each of the countries are nevertheless interested in protecting the interests of their domestic manufacturers of  $Z$ , but are barred from implementing explicit protectionist practices, they both publicly announce that they will execute a trigger strategy on the importing firm should it choose to participate in the home market. This trigger strategy, which

takes one period to implement, reverses the information advantage held by the foreign firm, requiring it to publicly announce its production quantity before arriving to the market. In other words, under the trigger strategy, the home government now requires that the exporting firm announce the quantity of  $Z$  that it will bring to the market, *first*. This reverses the Stackelberg ‘advantage’ that was once held by the foreign firm, as it is transferred over to the domestic firm.

This assumption is interesting and novel for several reasons. The first of these is that it has not been considered either in the literature or in practice, and thus makes for an interesting

### III.) Deriving the Equilibrium Strategies

The optimal strategy set for each firm is the strategy which maximizes the present-value of its expected payoff from the game. Although it is not integral to the results, the symmetry of the firms and markets in each country allows us to successfully solve this game by only considering the optimal strategy of the domestic firm. This implies that the sub-game perfect equilibrium strategy profile  $S_i$ , is the same for both firms. Since the set of strategies available to the two firm’s changes through time, as a result of both the government’s trigger strategy and the domestic firm’s perception of the foreign firm’s export ability, solving this game requires us to evaluate the payoffs from each period separately. To do this, we will first consider the expected payoffs from the first period.

#### 3.1 *The Sequential Game Assumptions*

In the first period of the game, the domestic firm has the option of either diversifying its business by exporting into the foreign market ( $D$ ), or producing only for the domestic market, which we denote as the isolationist strategy, ( $I$ ). Since the firm has the option of participating in two markets, we must consider the payoffs from participating in each of these markets separately from one

another, even though it is assumed that a firm will always produce in its domestic market.

The first market we will consider is the domestic firm's home market. Knowing that the foreign firm is free to export and sell in this market, the domestic firm has two sub-strategies available to it in the domestic market; it can either anticipate that the foreign firm will diversify, by producing the Stackelberg optimal quantity, which we denote as strategy  $S$ , or it can ignore the possibility, and produce the monopolist profit-maximizing quantity, denoted as strategy  $M$ . The Stackelberg and monopolist quantities are given below in (2) and (3), respectively:

$$(2) \quad Q_S^* = \frac{\alpha + \tau - c}{2}$$

$$(3) \quad Q_M^* = \frac{\alpha - c}{2}$$

Given these two production alternatives, we next need to consider the expected payoffs for each of them. Even though the level of output is notably higher under the Stackelberg equilibrium level of output, the effects of the elasticity of demand, which is defined in this case to be  $-p/q$ , as well as the best-responses of the foreign firm each play a significant role in defining the expected payoffs. Since the domestic firm arrives to the domestic market first, and perceives the foreign market's threat of successfully diversifying into this market with  $\{P = 0.5\}$ , the expected payoff for a strategy,  $i$ , is given by:

$$(4) \quad E[S_i] = (P)(\pi(\textit{foreign entry}) + (1 - P)(\pi(\textit{no entry}))$$

Using this methodology, we are able to derive the expected values for both ( $S$ ) and ( $M$ ). Solving for each of these expected values, we find that the optimal isolationist strategy, ( $I$ ), is ( $M$ ); to ignore the export threat by the foreign firm. We find that ignoring the export threat of the foreign firm is optimal so long as the following relation is satisfied:

$$(5) \quad \frac{1}{16}c^2 + \frac{1}{8}\alpha\tau - \frac{3}{8}c\tau + \frac{3}{16}\tau^2 > 0$$

Since two-way trade can only exist where  $\alpha > \tau$ , this relation holds true over the entire set of production possibilities, and therefore, the strategy of ignoring the foreign firm's export threat will always be the dominant strategy, so long as the probability of the foreign firm's threat succeeding is  $P(X = \text{Success}) \leq 0.5$ .<sup>7</sup> The expected payoff from strategy (I), in the first round is therefore found to be:

$$(6) \quad E[\pi_I] = \frac{3}{16}\alpha^2 + \frac{1}{8}\alpha\tau + \frac{5}{16}c^2 - \frac{1}{2}\alpha c - \frac{1}{8}c\tau$$

Now that we have defined the optimal strategy in the domestic market, we may next consider the optimal diversification strategy that the domestic firm could pursue. These calculations are simplified by virtue of the fact that the Stackelberg advantage rests with the domestic firm in this situation. The result of this is that the foreign firm's level of output reduces to a constant in the domestic firm's profit equation. In turn, the domestic firm will select its profit-maximizing level of output subject to whether the foreign firm chooses to expect or ignore the domestic firm's exportation threat. Therefore, the expected value of diversifying is the average profit of these two outcomes, plus the optimal production level in the domestic market, which we found to be (M), and is given as:

$$(7) \quad E[\pi_D] = \frac{3}{16}\alpha^2 + \frac{7}{32}c^2 + \frac{3}{16}\alpha\tau + \frac{1}{8}c\tau - \frac{9}{16}\alpha c - \frac{1}{32}\tau^2$$

A simple examination of (7) reveals that  $E[\pi_D] > E[\pi_I]$ . In fact, so long as the domestic firm is able to realize an economic profit when it diversifies, this relation will *always* hold since we assume that the domestic firm will adopt [D] when it can realize the home and foreign markets are independent of one another, and we assume that the domestic firm is *always* a producer in the home market. We may characterize this relation with the following functional form:

$$(8) \quad E[\pi_D] = E[\pi_I] + E[\pi_{\text{Export}}]$$

---

<sup>7</sup> As a result of this requirement, the equilibrium production strategy in the domestic market during the second and third periods is to produce the Stackelberg equilibrium quantity since  $P(\text{Success}) > 0.5$  in each of these periods.

Therefore, so long as  $[\pi_{Export}] \geq 0$ , which is the economic profit from exporting into the foreign market,  $E[\pi_D]$  will always yield a higher expected payoff. Although  $E[\pi_D] > E[\pi_I]$ , it may not yet be confirmed that the domestic firm will elect to export to the foreign market, as we must first evaluate the expected payoffs in each of the sequential periods to determine the sub-game perfect equilibrium strategy profile. In other words, in order to determine which strategy the domestic firm should adopt in the first period, we must determine which strategy generates the highest expected profit over the duration of the game.

### *3.2 The Second and Third Periods*

The methods for evaluating the payoffs for each of the strategies in the second and third periods is identical to that which was used to evaluate the first period payoffs, except that we must now consider the effects of government intervention. Since the third player in this game, the domestic government, will only implement its trigger strategy if the foreign firm does export into the Home market, it was not necessary to consider its role in the first period. However, now that we are evaluating the subsequent periods, we must consider the effect trigger strategy in both the Home and Foreign markets.

As a result of the introduction of these new possibilities, the set of payoffs that we must consider in evaluating the expected payoff at each decision node increases significantly. However, a simpler way around this exists due to the fact that each sub-game of this model is itself a one-shot game. This is a convenient nuance which arises from the Stackelberg market model that we are considering. Therefore, so long as we can show that (ii) above generates a positive payoff, ( $\pi > 0$ ), then we may deduce from the first iteration that diversifying is the equilibrium strategy in all subsequent periods. To test this, we compute the spread between the domestic firm's export payoff after the Stackelberg Advantage has been reversed, and the expected export payoff in the prior period. If this spread is non-negative,  $S > 0$ , then we have confirmed that diversification is the

equilibrium strategy in all subsequent periods, *ceteris paribus*. This spread,  $S$ , is defined as:

$$(9) \quad S = \left( \frac{P(\alpha + c - 2\tau)}{2} - \frac{P(\alpha - \tau + 2c)}{4} \right) > 0$$

This relation will always hold since we assume that partial quantities of commodity  $Z$  are not possible, we can confirm that diversifying will be the selected strategy in all subsequent periods of the game.<sup>8</sup> While this is a convenient result, the satisfaction of (9) has far greater implications, as it demonstrates that the government's prescribed trigger strategy in this Stackelberg market *incentivizes* the trade that it had sought to dissuade. This in turn begs the question, if the government's trigger strategy fails to deter imports of  $Z$ , what effect does the strategy have on the domestic firm whose interests it sought to protect?

### 3.3 The Effect of Government Intervention on the Domestic Firm

So just how effective is the government in protecting the domestic firm with this trigger strategy? The answer is not at all. We can demonstrate this by simply comparing the domestic profits before and after the trigger strategy was enacted. Letting  $\pi_A$  denote the domestic firm's profit before the trigger strategy and  $\pi_B$  equal the firm's profit after the strategy, we find that the firm's payoff is actually reduced as a result of the government's intervention:

$$(10) \quad \pi_A - \pi_B = \frac{3\alpha^2 - 14\alpha c + 2\alpha\tau - 6c\tau + 15c^2}{16} > 0$$

The reason for why the domestic firm is hurt as a result of its own government's intervention arises for no other reason than the foreign firm's newfound status as the market leader. This is a form of commitment power

---

<sup>8</sup> Although (9) implies that we do not need to value the payoffs during each period of the game, the precise calculations for each of the individual payoffs, which are derived through basic calculus, are provided in the Appendix. These calculations confirm the diversification strategy (D) suggested by (8).

because the foreign firm is unable to adjust its level of output following its announcement. This results with the foreign firm selecting its profit-maximizing level of output where the price elasticity of demand is most inelastic. Because of this, the domestic firm is forced to select its payoff-maximizing quantity in the relatively more elastic regions of the market, which results in it selecting a lower quantity of output than it did when it was the market leader. Unfortunately, even though the market price for  $Z$  will be higher after government intervention, it will not be sufficiently high enough to offset the revenue lost from the reduction in output.

#### IV.) The Nash Equilibrium: An Optimal Strategy Profile

Having established in (9) that diversifying (D) *always* results in a higher expected payoff with government intervention than without, and having established that diversifying (D) in the first period yields the highest payoff, we have shown that the best strategy profile available to the domestic firm is to diversify in every period:  $S_D(x^*) = \{D, D, D\}$ . By symmetry, we can dually conclude that this is also the best strategy profile available to the foreign firm as well:  $S_F(x^*) = \{D, D, D\}$ . Since these are the best strategy profiles available to either firm, we can conclude that these profiles comprise the game's subgame perfect equilibrium. In other words, the Nash Equilibrium at each point in this super-game is to export into the other firm's home market (D). We may formally define this equilibrium point as:

$$(10) \quad S(x_D^*, x_F^*) = S((D, D, D), (D, D, D))$$

Although the equilibrium strategies in the first period are suboptimal, the subgame perfect equilibrium has the double identity of also being the optimal strategy set of the game. How this peculiarity arises is the sole result of the trigger strategies implemented by the home and foreign governments. As conveyed by (9), the governments' trigger strategy, by no mere coincidence, incentivizes imports by offering exporters a higher level of profit than in either

autarky or the unregulated market. Because of this, firms have an incentive to initiate the trigger strategy of their export market's government at the first possible opportunity, which is the first period of the game, as it is this strategy that opens the door to the highest potential profits. In addition, it is worth noting that despite the adverse effects that the government's intervention produces for its domestic firm in the partial equilibrium setting, the coordination failure between these two countries and firms actually results in a general equilibrium that makes both firms and both countries substantially better off. Furthermore, since it can be shown that this result holds true in both Cournot and autarkic markets, a more thorough analysis of this form of cooperative information coordination, by governments, can and should be completed. This is especially true in the context of international trade agreements, as it is readily apparent that there are significant welfare gains to be realized from this cooperative approach.

## V.) Concluding Remarks

Using a finite sequential game, this chapter develops considers a special case of reciprocal dumping, in which a protectionist practice enacted by governments has the effect of reducing the welfare of the domestic industry it sought to protect. This is augmented by the fact that the government's trigger strategy of regulating the market goes into permanent effect once it observes the arrival of a foreign firm in the country's market, the trigger. There is no question that this is a bold assumption, as it naturally raises the question, *why would the government continue to regulate a market when domestic producers incur a loss in profit as a result?*

Despite the seemingly pointless nature of this government intervention, there are two circumstances in which the continued regulation of this market could

prove to be efficient.<sup>9</sup> The simplest of these instances occurs when the cost of developing trade policy and any other legislation, or for that matter, reversing trade policy and other legislation, exceeds the losses incurred by the domestic firm. This is an entirely possible scenario, and one which can be efficiently resolved by requiring that the government compensate the domestic firm for its losses. The second instance in which the prolonged execution of this trade policy is efficient arises exists when the improvements to the country's welfare, on the whole, exceed the losses of the firm. Unfortunately, this cannot occur given our assumption of a Stackelberg market, but could however be readily applied when the unregulated market is a Cournot duopoly.<sup>10</sup> This situation could feasibly arise under this market structure owing to the fact that the consumer surplus of the domestic consumers is considerably higher after government intervention than in the unregulated Cournot market.

Perhaps the most interesting conclusion reached during the course of this analysis has been the subgame perfect equilibrium's dual personality as the optimal strategy profile of the game. While this is intriguing for a host of reasons, the fact that this arises as a product of the governments' use of permanent trigger strategies features most prominently. This is of interest because the trigger strategies, despite being *dominated* strategies, when jointly executed have a certain synergy that significantly enhances the profits for both firms in the model. However, because the trigger strategy is itself dominated, the equilibrium we find in this game is unlikely to emerge in a sequential environment where the trigger strategy may be reversed. This is admittedly unfortunate, as we find that when the requirement by both governments that exporting firms announce their export quantities in advance, the profits of two-way trade are *substantially* higher than in any other possible market structure.

---

<sup>9</sup> We define efficiency through a Kaldor Criterion, which asks us to simply consider whether a Pareto optimal outcome can be reached if those that gain under the prolonged existence of the government's intervention *could* compensate the parties which are harmed, and still be better off.

<sup>10</sup> It is worth noting that if we were to remodel this game substituting our Stackelberg market with a Cournot, we would still arrive at the same subgame perfect equilibrium.

## Conclusion

We have aptly characterized the optimal strategies of firms engaged in two-way trade, in two unique settings. In Chapter 2, where we analyze the static case of a duopoly facing an asymmetric cost structure that is engaged in two-way intra-industry trade. Through our analysis of this instance, we show that the asymmetric nature of the firms involved gives rise to dumping, with the firm with the lowest economies of scale dumping in its export market. Additionally, we also consider in our analysis the existence of arbitrage opportunities in instances where dumping is prevalent. Although we rely on several assumptions to generalize these instances in assuming that there are minimal fixed costs associated with arbitrage, the existence and effect of these opportunities is worthy of a further analysis. This is but one of several possible extensions that may be made to the framework we present in Chapter 1, though none should feature more prominently than an analysis of the income distribution effects from such trade. Although time constraints prevented us from being able to analyze these effects, such an analysis is integral to determining whether or not two-way trade is in fact beneficial to both economies.

In Chapter 3, we develop and characterize the optimal strategy profile of two firms engaged in intra-industry trade in a repeated game environment. Without question, the most significant results emanating from this Chapter is our conclusion that trade regulation through export coordination between countries engaged in intra-industry trade offers substantial improvements in profitability throughout the industry. At the same time, because the market price in both markets decreases as a result of government intervention there is a corresponding increase in consumer welfare. While this result is indeed significant, the possibility that this coordination could have an adverse effect on the *overall* welfare of both countries is within the realm of possibilities. For instance, even though consumer surplus is greatly improved when coordination is enacted under both autarky and Cournot duopoly settings, this proves not to be the case of the

Stackelberg market that we originally considered in our sequential game. As a result, our analysis presented in Chapter 2 is still incomplete, since our ability to thoroughly generalize the circumstances in which export coordination is an optimal strategy, hinges on our understanding of the income distribution effect it has on an unregulated Stackelberg environment. Nevertheless, we are able to identify at least two common market environments in which governments' coordination of industry information can improve the welfare of not only the industry itself but the country, entire.

In conclusion, this paper characterizes two unique scenarios in which intra-industry trade can give rise to dumping. The noted differences between these two scenarios have enabled us to better analyze the limitations and generalizations that are made with the Brander-Krugman [1] model in both static and sequential settings. However, this is by no means the most significant contribution this paper makes to the literature. Rather, our demonstration of the effects of government coordination of inputs has a wide range of applications on the policy-making level, as the welfare of countries who agree to adopt this form of regulation is significantly improved. An obvious extension of the results from the characterizations we present in this compilation that can and should be made is an analysis of the income distribution and welfare effects that emanate from each of these characterized environments.

## References

- [1] Brander, J., Krugman, P.: A ‘Reciprocal Dumping’ Model of International Trade. *Journal of International Economics* **15**:313-321 (1983).
- [2] Brown, W., Hogendorn, J.: *International Economics in the Age of Globalization*. Toronto, ON: University of Toronto Press (2000).
- [3] “Five Years After WTO, Trade Disputes Increase.” *China Daily*. 20 December 2006. <<http://www.china.org.cn/english/BAT/193149.htm>>. (14 May 2009).
- [4] Krugman, P., Obstfeld, M. *International Economics: Theory and Policy*. New York: Addison-Wesley, 2006.
- [5] Pinto, B.: Repeated Games and the ‘Reciprocal Dumping’ Model of Trade. *Journal of International Economics* **20**:357-366 (1986).
- [6] Trebilcock, M.: *The Regulation of International Trade*. London, UK: Taylor & Francis, Inc. (1999).
- [7] “U.S. to impose 24.75% anti-dumping duty on Chinese off-road tires.” *Gasgoo*. 25 February 2008. <<http://autonews.gasgoo.com/auto-news/5804/U-S-to-impose-24-75-anti-dumpling-duty-on-Chinese-off-road-tires.html>>. (13 May 2009).

*Appendix*

---

## Appendix to Chapter 2

---

### A Simple Trade Equilibrium Example Assuming Asymmetric Costs:

Assumptions:

- (1) The Demand function is linear, and is defined as:  $q_d = 20 - p$
- (2) The transaction cost multiplier,  $\tau$ , is 1.1, which implies:  $g = 0.9$
- (3) The foreign firm's marginal cost is  $c = 3$

The total revenue functions for the domestic and foreign firm are respectively given as:

$$\pi_H = (20 - (x + y))x + (20 - (x^* + y^*))x^* - (x + x^*)^2 - x^*/g - F$$

$$\pi_F = (20 - (x^* + y^*))y^* + (20 - (x + y))y - c(y + y^*/g) - F$$

Taking first derivatives of the total revenue functions, we arrive at the following marginal revenue functions:

$$\pi_H = 20 - 2x - y + 20 - 2x^* + y^* - 4x - 4x^* - 1.1 = 0$$

$$\pi_F = 20 - (x^* + 2y^*) + 20 - (x + 2y) - c - 1.1 = 0$$

With a little rearranging, this system can be brought into the matrix notation,  $AX = B$ . This gives a 4x4 matrix, which encompasses the marginal revenue functions of both markets. Using Cramer's rule, we arrive at the following equilibrium levels of output:

$$q_H^* = 0.6316$$

$$q_H = 1.7316$$

$$q_F = 7.0842$$

$$q_F^* = 8.1842$$

The equilibrium price is:  $P = 11.1842$ . The law of one price is maintained.

Deriving the Marginal Revenue functions:

In chapter 2, we derive the marginal revenue functions using a method to simplify our calculations, by expressing marginal revenue in terms of  $\varepsilon$  and  $\sigma$ . This section illustrates how this substitution was made:

Beginning with the original total revenue function, and letting  $y$  be foreign output, we have:

$$\pi = p_H(x + y)x$$

Because of our symmetry assumption, we need only consider a single market. By the product rule, and letting  $z$  represent total output, we arrive at the following marginal revenue function:

$$d\pi = p(z) + x * \frac{dp(z)}{d(z)}$$

Since this is not a particularly easy function to generate reduced form equations with, we multiply the right hand side by  $(z/z)$ :

$$d\pi = p(z)[1 + \frac{z}{z} * \frac{x}{p(z)} * \frac{dp(z)}{d(z)}]$$

With a little inspection, it is clear that this equation contains the inverse of the point elasticity of demand  $(z/p)*(dp(z)/d(z))$ :

$$d\pi = p(z)[1 + \frac{x}{z} * \frac{z}{p(z)} * \frac{dp(z)}{d(z)}]$$

Rearranging, we get:

$$d\pi = p(z)[1 - \frac{x}{z} * \frac{1}{\varepsilon}]$$

Because  $(x/z)$  represents the market share of the domestic firm, we can use the complement to represent the foreign market share,  $(1 - (x/z))$ , but first, we must remove the inverse of the elasticity of demand. This gives:

$$d\pi = p(z)[\varepsilon + (1 - \sigma)]$$

This is identical to the total revenue function for each firm in its home market.

Comparative Static Analysis (An Export Subsidy Application):

To generate our comparative-statics for the market equilibrium, we will have to first consider the simultaneous system of equations that make up each of the two markets. Each of these systems is defined by the functions that define the profits of each the duopolists in the specified market. Solving this system for our variable of interest,  $\sigma$ , necessitates satisfying the sufficiency conditions of the Implicit Function Theorem, which we will aptly demonstrate by showing that the Jacobian determinant,  $|j|$ , of all our comparative-static systems are non-zero.

The first comparative-static derivatives that we shall consider involve the change in  $\sigma$  with respect to changes in export production. There is significant merit in analyzing this comparative-static due to its numerous applications to matters of international trade policy. One particular extension that readily comes to mind is the case of governmental export subsidies. For instance, suppose the domestic government were to begin to subsidize the domestic firm's export business as part of a larger policy to improve its presence in international markets. This subsidy, which for the sake of simplicity is assumed to be in any form other than an ad-valorem subsidy, will, so we assume, successfully incentivize the domestic firm to increase its level of export production. Given the possibility of such a program, especially in light of programs such as the United States' Export Enhancement Program (EEP), it is naturally advantageous for the foreign firm to analyze how the relevant market will adjust, *ceteris paribus*, given the domestic firm's increased level of exports. It is with comparative-static analysis that this can easily be achieved. For simplicity, and without loss of generalizability, we use the comparative static of  $\sigma_F^*$  to analyze the effect of the domestic government's implementation of an export subsidy. The relevant comparative static derivatives under this scenario for the domestic and foreign markets are respectively as follows:

$$\frac{\partial x}{\partial y} = \frac{pxc}{(-pxc - 2\epsilon x^2 c - 4\epsilon xyc - 2\epsilon y^2 c)} < 0$$

*WLOG*( $g = 0$ )

$$\frac{\partial \sigma_F^*}{\partial x^*} = \frac{4x^2 + 2c + 2x\epsilon + \frac{4\epsilon}{g} - 4x}{(2g\gamma + cg + 1)^2} > 0$$

The respective signs on each of the comparative-static derivatives confirm the stability of each of the trade equilibriums. They also provide us insights into how the model returns to an equilibrium following an exogenous increase in exports, *ceteris paribus*. Economically speaking, this enables us to be able to better understand the rate at which the domestic firm must readjust its domestic output in order to return the market to equilibrium, given this exogenous increase in exports.

## Appendix to Chapter 3

---

An Analysis of Equations (4) and (5):

Equation 4:

$$E[S_i] = (P)(\pi(\text{foreign entry}) + (1 - P)(\pi(\text{no entry}))$$

This defines a firm's expected profit in its home market for a given strategy. Equation 4 is the expected average of two possible outcomes for a given strategy. These two outcomes are:

- (1) The other firm is successfully able to enter its rival's home market
- (2) The other firm is unable to enter its rival's home market

This is a weighted average because a firm perceives that the other firm will successfully enter its home market with  $(P_n = .25(n + 1))$ , where  $n$  denotes the number of periods. The optimal strategy for a firm is to choose the strategy that maximizes its expected profit.

Equation 5:

$$\frac{1}{16}c^2 + \frac{1}{8}\alpha\tau - \frac{3}{8}c\tau + \frac{3}{16}\tau^2 > 0$$

This defines the profit spread of a firm behaving as a monopolist versus a Stackelberg Leader in its home market in the first period. In expected profit notation, and using the subscripts M and S to denote the monopolist and Stackelberg leader profits, respectively, this can be expressed as:

$$E[\pi_M] - E[\pi_S] > 0$$

The intuition behind this equation is that if a firm's expected profit from behaving as a monopolist, exceeds the expected profit from behaving as a Stackelberg leader, then the firm should produce the monopolist quantity of output. However, because the firm perceives that the other firm will successfully enter its home market with  $(P_n = .25(n + 1))$ , the producing the monopolist quantity of output is only optimal in the first period after the game.

*Solutions to the Sequential Game Tree*

Period 1 (Optimal Strategy at Home is to Produce Monopolist Quantity):

1.1 (Diversifying):

$$\pi = \frac{3}{16}\alpha^2 - \frac{9}{16}\alpha c + \frac{7}{32}c^2 + \frac{3}{16}\alpha\tau + \frac{1}{8}c\tau - \frac{1}{32}\tau^2$$

1.2 (Isolated):

$$\pi = \frac{3}{16}\alpha^2 + \frac{5}{16}c^2 - \frac{1}{2}\alpha c + \frac{1}{8}\alpha\tau - \frac{1}{8}c\tau$$

Period 2 (Optimal Strategy at Home is to Produce Stackelberg Quantity):

2.1 (Diversifying in both periods):

$$\pi = \frac{71}{128}\alpha^2 - \frac{1}{4}\alpha^2 - \frac{13}{16}\alpha\tau + \frac{27}{128}c^2 + \frac{33}{64}\tau^2 - \frac{7}{32}c\tau$$

2.2 (Diversifying only in period 2):

$$\pi = \frac{55}{128}\alpha^2 - \frac{13}{16}\alpha c + \frac{19}{64}\tau^2 + \frac{103}{128}c^2 - \frac{27}{32}c\tau$$

2.3 (Isolated in Period 2):

$$\pi = \frac{23}{128}\alpha^2 + \frac{3}{16}\alpha\tau - \frac{1}{2}\alpha c - \frac{7}{32}c\tau + \frac{43}{128}c^2 + \frac{1}{64}\tau^2$$

Period 3 (Optimal Strategy at Home is to Produce Stackelberg Quantity):

3.1 (Diversifying and diversifying prior to period 3):

$$\pi = \frac{71}{128}\alpha^2 - \frac{1}{4}\alpha c - \frac{9}{16}\alpha\tau + \frac{27}{128}c^2 + \frac{45}{64}\tau^2 - \frac{21}{32}c\tau$$

3.2 (Diversifying only in period 3):

$$\pi = \frac{11}{16}\alpha\tau - \frac{3}{64}\tau^2 - \frac{5}{32}c\tau + \frac{23}{128}\alpha^2 - \frac{1}{2}\alpha c + \frac{43}{128}c^2$$

3.3 (Isolated in period 3):

$$\pi = \frac{23}{128}\alpha^2 + \frac{7}{16}\alpha\tau - \frac{1}{2}\alpha c - \frac{21}{32}c\tau + \frac{43}{128}c^2 + \frac{13}{64}\tau^2$$

