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An Agent-Based Network Simulation Model for Comprehensive Stress Testing and Understanding Systemic Risk

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Abstract

This paper develops an agent-based network simulation model that measures systemic risk in the U.S. banking system. It is shown that ultimate losses to a bank after an initial shock to the system is greater than the direct loss they would expect to face. Using actual balance sheet data and simulating over randomly-generated interbank networks, the model captures the feedback effects that arise from a shock to the highly connected and interdependent system. In addition to capturing these "extra" losses that arise from bank interactions over several periods, this framework measures the different channels through which the initial risk propagates, amplifies, and transforms. The model is then employed in Monte Carlo simulations for stress tests, which are analyzed from both the perspective of a bank risk manager and a regulator. It is also implemented in a Value-at-Risk framework to demonstrate its potential to inform existing VaR models employed by banks. An important feature of the banking system that is often omitted in related models is collateral underlying the majority of interbank transactions. The simulations reveal that as much as 30% of total losses due to an asset shock are due to strains in the repo market. The regulator stress tests highlight the extra risk faced by banks heavily involved in the securities and interbank markets. A wide variety of different scenarios can be tested in this framework, and by collecting detailed information throughout the simulation, the composition of systemic risk and its evolution through the system can be analyzed in different ways.

1 Introduction

A benefit of a well-developed financial system is that different risks faced by individuals, corporations, and governments can be broken down into pieces and allocated efficiently across time and space. Financial intermediaries facilitate this process by developing and managing financial instruments and markets that allow this allocation to happen. An individual's pension savings can be invested in a foreign real estate market as easy as it is to leave it in a savings account, for example, and the interconnectedness of the global financial system fosters this liquid allocation of risk and capital. However, as financial crises have revealed in the past, with this connectivity comes a very high propensity for these risks to change and propagate through the system in times of crisis. Central to these systemic crises are banks, which both retain many of these risks and serve as conduits through which risk flows. Despite the sophistication of their risk-management systems, they are often caught off-guard when disaster strikes. A primary reason for this is that their tools such as Value-at-Risk (VaR) models and stress tests are based on historical data, which is of no use when new problems arise, the propensity of which is increasing rapidly with recent financial innovations.

The relationships between institutions in the financial system are as important as the institutions themselves, motivating a growing body of research that models the financial system from a network-theoretical approach. This approach is natural when trying to understand the nature of systemic risk, as the risk of contagion depends on the linkages between nodes in the network. Although much can be gleaned about a system by studying its network structure, the financial system is also a complex, adaptive system (CAS) as it is composed of many heterogeneous participants who behave in different ways in response to ever-changing conditions they face as the system evolves second-by-second. There is a constant feedback process in the system so that even if one could predict how an institution will react in one situation, their action will change the environment and influence the decisions by others, further altering the state of the system. Modeling phenomena like this is the goal of agent-based models (ABMs), which allow for the emergence of macroscopic[1] system-wide dynamics from the autonomous actions by the individual agents comprising the system that would otherwise be intractable analytically.

Especially in times of financial turmoil it is very difficult to predict what will happen next since the selfish decisions of the different institutions can transform non-linearly into system-wide changes, which triggers further autonomous action. The model described here aims to synthesize the network and agent-based approaches to study the nature of systemic risk in the financial system. The main contributions of this model are described in detail in section 1.2, but summarized below. The model

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[1]Bookstaber, 2012

- Includes risk dynamics driven by collateral in the interbank debt markets, a factor that has been largely omitted from existing research.

- Measures the contribution of different channels that comprise overall systemic risk. It allows for a summarized breakdown of the different channels from a Monte Carlo simulation as well as detailed evolution of the channel contributions through time in single "runs" of the simulation.

- Allows for detailed analysis of these channels from the perspective of a bank risk manager as well as regulators, applied in a stress-testing and VaR framework

1.1 Literature Review

There is an increasing number of studies that investigate systemic risk using network models, a good survey of which can be found in Upper and Worms (2011). This approach has also been used extensively by central banks (Cont et al., 2012) to assess default and contagion risk in their respective financial systems. Some of the earliest work in this area was by Allen and Gale (2000), who showed the dependence of contagion on the particular structures of the interbank network. Simulation studies using this approach have been used to allow for the testing of a wide variety of network structures, assumptions, and counterfactuals, shedding light on how certain properties of the system increase the chance of contagion (Elsinger et al, 2006, Upper, 2007). There is not a well-defined consensus regarding the importance of default contagion–the risk that a default of one bank will cause a "domino effect" – for systemic risk as some empirical studies found that in most cases default contagion is not very important (Elsinger et al., 2006, Upper and Worms, 2004), while others found that it plays a large role (Cont et al., 2012).

The majority of papers in this area focus on interbank contagion triggered by a defaulted bank, and make conclusions about the network structure as it relates to contagion. Cont et al. (2012) extend this methodology to the Brazilian banking system, which has rich data on balance sheet compositions and bilateral exposures. They study different institutional and network characteristics that affect the systemic importance of individual banks, and differentiate between fundamental defaults and defaults caused by contagion. A related approach of dissecting further the different ways banks can suffer losses is the consideration of the different ways that contagion can start in a banking system. Chan-Lau (2010) identifies four types of shocks to a balance sheet that can trigger spill-over effects in the banking system: Credit shocks, funding shocks, risk transfer shocks, and off-balance sheet exposures. Employing a balance sheet-based network analysis approach makes this flexibility possible as each shock affects the balance sheet in different ways that can result in a different transmission of risk to other banks.

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The model presented in this paper is a type of "balance sheet-based" network model and is based on the framework developed in Montagna and Kok (2013), who employ an agent-based approach to study systemic risk in the EU banking system. They also use a "multi-layered" interbank network that captures a few different ways that banks are interconnected. The three layers are short-term bilateral exposures, long-term bilateral exposures, and common exposures to financial assets. They show that when activating all three exposure networks, the total risk to the system is greater than the sum of risks when the single layers are activated individually. They generate random interbank networks guided by balance sheet and high-level data on cross-country credit exposures. The structure of the network is their degree of freedom in the simulation, and aim to identify relationships between contagion and network structure. They also construct an algorithmic measure of the systemic importance of banks within the system. In addition to simulating a contagion situation where a "trigger bank" defaults, they consider a shock to the banking system in the form of a large market value decline to a widely held asset class, which they show has a greater effect on systemic risk, captured by the many shared exposures banks have to the shocked security.

The model described here is an extension of this framework, where some assumptions are relaxed, behavioral rules altered, and new layers added to the interbank network. The first major deviation from Montagne and Kok (2013) is the inclusion of collateral in the interbank markets. Much of interbank lending/borrowing is in the form of repurchase agreements (repo), where the borrower "sells" securities to the creditor bank while agreeing to buy them back at an agreed-upon later time for a little more than they sold them for, which is the effective interest rate on the loan. In other words, it is a loan collateralized by securities. The repo market played a very important role in the recent financial crisis and was a primary source of systemic risk and the amplification of the mortgage market collapse. There was a marked mismatch between many banks' assets and liabilities as they relied heavily on the cheap, short term funding in the repo market to take positions in longer-term, less liquid assets. When uncertainty about the financial health of banks and other financial institutions with large exposures to the housing market, primarily in the form of intrinsically leveraged structured products and derivatives, the repo market froze, reducing access to the form of funding they all depended on. In addition to this funding strain, the myriad varieties of mortgage-backed securities were also used as collateral for much of this credit, prompting margin calls that precipitated further strain on the system.

Including this dynamic in the model allows for a different transmission of losses to a creditor bank in the case that a debtor defaults. Many of the network models described above assume different "loss given default (LGD)" levels, while others, including Montagna and Kok (2013), use Eisenberg and Noe's (2001) "fictitious default algorithm," which assumes that upon a default of a

bank, creditors to that bank are compensated with an equal share of the liquidation value of the delinquent banks assets. They show that their algorithm accomodates "instantaneous" contagion since it ensures that the system settles to the same unique state that it would if the banks took action simultaneously. In this model, when a repo counterparty[2] defaults, the creditor sells the collateral assets to make up for their lost cash. In the short time-span and crisis scenarios that this model is concerned with, this approach reflects observed bank behavior[3], and aims to capture simultaneous behavior algorithmically.

The importance of the repo market as a source of risk as well as an amplification channel for contagion is highlighted in Duarte and Eisenbach (2013), Greenwood et al. (2012), and Begalle et al. (2013). Duarte and Eisenbach use an empirical model based on the theoretical model developed by Greenwood et al. to measure the vulnerability of the U.S. banking system to "fire-sale spillovers from an asset shock, and categorize losses into different categories: Initial shock, direct losses, asset sales, price impact, and spillover losses. These measurements are linear and do not allow for the autonomy and multiple-time period dynamics. The model presented here incorporates this approach of breaking down the systems risk into different channels, but is able to do so in an agent-based simulation framework. Begalle et al. (2013) estimate the risk of fire sales in the tri-party repo market, whose findings inform parameters used in the model described below.

1.2   Contribution

Montagna and Kok (2013) significantly contributed to the network simulation approach to studying systemic risk in the banking system, especially by incorporating an agent-based approach that is ideally suited for this kind of problem. Using a framework based their approach, this model contributes to this body of research in a few ways. First, by relaxing some parameter assumptions used in Montagna and Kok and replacing them with data-based estimates, we can observe dynamics that may be closer to those seen in reality. Second, because satisfactory data on long-term bank debt was not available, and it has been shown that off-balance sheet over-the-counter derivatives plays a large role in amplifying systemic risk, this model replaces the long-term interbank exposure network with an off-balance sheet OTC derivative exposure network.

A major feature of the model not employed in a network framework is the collateral channel that underlies much of the interbank obligations in the banking system. Despite providing protection for creditors against defaulting counterparties, these arrangements have been shown to sometimes exacerbate issues as a source of fire sale risk.

A primary goal of this model is to be able to differentiate between the different channels through

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[2]In this case the creditor enters a "reverse repurchase agreement"

[3]See Begalle et al., 2013

which shocks propagate over the course of the simulation. The model can make sense of the detailed data collected throughout the simulation, providing a richer story of how different channels contribute to overall risk, but also how these channel contributions interact and change over multiple periods. A common drawback of agent-based models is that they tend to be "black boxes" that take inputs and produce results with very little enlightening information in between. This model aims to open the black box with careful data collection, interpretation and presentation, that can provide useful information for banks and regulators who want to better understand the reasons why they are vulnerable to certain shocks and how that vulnerability arises.

Many related models examine systemic risk from the perspective of a regulator who is interested in the vulnerability of the system to certain shocks and to what degree individual banks and their interconnections contribute to this vulnerability. An important feature of this model is its application to individual bank-level risk management. Performing stress tests with this model can shed light on the feedback effects of an initial shock to the banking system, allowing them to better anticipate where losses will come from. In addition to stress tests, this framework can be used in a VaR analysis setting. An agent-based simulation approach to VaR models and stress tests decreases their reliance on historical data, and by explicitly monitoring the propagation and transformation of a shock as it moves through the system, can give a more complete view of a banks risk. Even if a stress test accounts for correlations between security types in their portfolios, assessing the expected loss due to a price shock alone may not tell the whole story. A simple price drop of an asset class can turn into funding problems and spur fire sales in unexpected markets. Likewise, a counterparty default can have farther-reaching effects than just counterparty credit risk. In cases like these, autonomous actions by agents alter the environment, which causes further action, and on aggregate can lead to unexpected consequences to the system.

1.3   Model description and outline

The financial system in the model consists of several randomly generated interbank networks and ten banks with heterogeneous balance sheets based on regulatory data from Q4 2013. At the beginning of each "trial" of the simulation the system is shocked either by an asset price decline or a bank default and each bank takes independent action to stay solvent. Their actions may have direct or indirect impacts on others, which will necessitate another iteration of individual bank decision-making in response to the changed environment. This process repeats until the system reaches an equilibrium where they do not have any more needs to fulfill. The next section describes the model set-up and dynamics, followed by a description of its use within the Monte Carlo simulation framework. The data and its mapping to the model is then discussed, followed by the results section. The model is employed to investigate the properties of systemic risk in

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response to a crisis scenario of large market value declines of a few asset classes, examined from both the perspective of a bank risk manager and a regulator. The model is then integrated into the traditional VaR analysis that banks currently use extensively for risk-monitoring (Bookstaber, 2012).

2   Banking system set-up

2.1   Balance Sheets and Initial Conditions

Bank balance sheets are based on information contained in Form FR Y-9C, consolidated according to the data mapping described in Section 4. As part of each bank i's assets a_i, they hold cash c_i, securities v_i, short-term interbank loans/repo d_i^s and other assets o_i^a. Their liabilities comprise deposits p_i, short-term interbank borrowing b_i^s, and other liabilities o_i^l. Although not explicity accounted for on their balance sheet, bank i also has off-balance sheet over-the-counter (OTC) derivative contracts with other banks, denoted by OTC_i.

$$a_i = v_i + c_i + d_i^s + o^a \tag{1}$$

$$l_i = p_i + b_i^s + o_i^l \tag{2}$$

Their security holdings v_i are composed of six different types of securities s_k^i where $k \in \{1, 2, 3, 4, 5, 6\}$ and $v_i = \sum_{k=1}^{6} s_k^i p_k^t$, $s_k^i, p_k^t > 0$ and p_k^t is the price of security k at time t. The six groups are

> 1) U.S. Treasuries (UST)
>
> 2) U.S. Government Agency and State securities (USGA)
>
> 3) Mortgage-Backed Securities (MBS)
>
> 4) Asset-Backed Securities and Structured Products (ABS)
>
> 5) Equity securities
>
> 6) Other debt securities

Each security type has a parameter α_k that measures the liquidity of the market for that security. The lower the value of α_k, the more liquid the market is, which has important implications in a firesale scenario since it dictates how elastic a security's price is to selling in the market.

Each security also has a haircut h_k, which is how much a borrower must commit above the amount of a collateralized loan, expressed as a percentage. Based on the FRBNY's tri-party repo

data, we assume that haircuts for all securities are fixed at 0.05[4].

Depending on the haircut h_k of security k and how much bank i has borrowed in the STIB market, a portion of their holdings s_k^i will be pledged to their creditors as collateral for their repurchase agreements. In this case the securities in custody remain on the bank's balance sheet but they are not available for immediate sale. Thus if a bank wants to liquidate some securities to stay solvent in the model, they are only allowed to sell securities not being used as collateral at time t. Differentiating between these two sets of securities, we define

$$s_k^i = s_k^{i,a} + s_k^{i,ua} \tag{3}$$

where

$$s_k^{i,a} = s_k - \Sigma_{j=1}^N m_{ji}^{sc} \tag{4}$$

is the amount of security k available for sale, and

$$s_k^{i,ua} = p_\kappa(m_{ji}^s(1 + h_\kappa)) \tag{5}$$

is the unavailable portion. h_κ and $\kappa = m_{ji}^c$

Similar to the securities set up, a portion of bank i's cash will also be tied up as collateral for various exposures. First, when banks face margin calls due to falling market values of their securities held as collateral against borrowing, they will post cash as margin to satisfy their creditors. Each bank also has off-balance sheet credit exposure to OTC derivative counterparties, the majority of which is collateralized by cash (USD and foreign currencies). Thus when trying to meet their own solvency requirements as well as immediate demands from creditors during the stress scenario, a portion of their cash holdings will be unavailable to spend. So $c_i = c_i^a + c_i^{ua}$, where c_i^a is their disposable cash and c_i^{ua} is the nondisposable portion.

2.2 Interbank networks

Banks are connected to each other through three different networks in the model, which are an integral part of the system's initial state and evolution. Their exposures and relationships with other banks will drive much of the behavioral dynamics of the system following an initial shock. Detailed data on these interbank linkages in the U.S. does not exist, so these network structures will be randomly generated and mapped to the model, a detailed discussion of which can be found in Appendix A.

The first network is the short-term interbank lending (STIB), or repo market, which is denoted by m^s, where m_{ij}^s represents a collateralized loan from bank i to j. This market is a major source

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[4]Discussed in Section 4

of funding for many banks and will play a large role in the propagation and transformation of risk during the simulations.

Underlying each transaction captured in the entries of m^s are collateral securities posted by the borrowing bank. The matrix containing this information is denoted by m^c, where each entry $m_{ij}^c \in \{1, 2, 3, 4, 5, 6\}$ in the matrix m^c of collateral asset types corresponds to the type of security held against a loan from bank i to j. This network comes into play when banks assess the availability of securities to liquidate since a portion of their holdings will be held in custody by repo counterparties. It will also be a potential source of fire sales as creditors will sell collateral held against loans to banks that default.

Banks are also exposed to each other through the off-balance sheet over-the-counter derivatives market. The network m^{obs} captures counterparty risk in the large OBS OTC derivative market, which, due to the inherent leverage in derivative transactions and their existence "off-balance sheet" is an important channel through which contagion can spread that is much larger than balance sheet sizes in the system would suggest. Each entry m_{ij}^{obs} of this matrix represents the off-balance sheet credit exposure bank i has to j. This network will come into play when bank's assess their available cash to spend as some of it will be used as collateral for their OBS OTC derivative counterparties, and in the event of a default, its effect on counterparties depends on this exposure matrix.

Another important network in the financial system is the network of common exposures to securities. As in Montagne and Kok (2013), consider a matrix m^v where each entry m_{ij}^v represents the percentage amount of shared securities between bank i and j's asset holdings. In other words, it is the degree to which a shock to the market price of an asset held by two different banks will affect the overall system. These exposures are implicit in the model as banks' shared exposure to assets is a consequence of their balance sheet compositions, but this shows that a connection between two financial institutions does not have to be an explicit transaction, and is an important dimension to the overall structure and dynamics of the system.

2.3   Bank behavior and contraints

Banks face a number of constraints that dictate how they react to shocks to their balance sheets and actions by counterparties. They must keep their risk-weighted capital ratio (RWCR) above the minimum requirement $\bar{\theta}$ of 8%[5]. Their RWCR is determined by

$$\theta_i = \frac{a_i - l_i}{w^{ib} d_i^s + \Sigma_{k=1}^6 w_k s_k^i p_k^t + \theta^o + w^{obs} OTC_i} \tag{6}$$

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[5]Basel III, "Overview of Capital Requirements in U.S. Basel III Proposals, Davis Polk & Wardwell, LLP, 2013

where w^{ib} is the risk-weight given to short-term interbank loans, w^k is the risk-weight of security k, θ^o is the risk-weight of the remaining assets, and w^{obs} is the risk-weight of off-balance sheet OTC derivative contracts. Cash c_i carries a risk-weight of zero.[6]

They must also maintain a simplified version of a minimum liquidity buffer to remain solvent. We assume that their only requirement is that they hold a minimum amount of cash against their potential cash outflows. In this case we focus on deposits and short-term borrowing, which are subject to relatively quick outflows in stress scenarios. Thus they must ensure that

$$c_i \geq \beta(p_i + b_i^s) \tag{7}$$

where β is the buffer size. We will fix β at 5% as in Motagna and Kok (2013), and consistent with deposit runoff and cash outflow assumptions in Basel III[7].

It is assumed that banks do not face withdrawals from their deposit liabilities p_i or their other liabilities o_i^l. On the asset side, only the "other assets" o_i^a are assumed to be fixed throughout the simulation.

3   Model dynamics

This section describes the model dynamics during a single run of the Monte Carlo simulation, which evolves over several "periods" t. Although important structural features of the environment in which the agent banks interact in the model such as the interbank exposure network and collateral matrix are randomly generated, once the exogenous initial shock is applied, the resulting dynamics are ultimately deterministic. In an agent-based framework it is difficult to predict the system outcome in response to a shock, but the constraints and hard-coded behavior of the agents guide the system to a unique equilibrium. Indeed, one of the primary aims of this model is "opening the black box" and creating a detailed and clear picture of its complicated dynamics. Although each trial lasts several "time-periods", the time-frame of this model in the real world would would be several hours to a few days. This is because it aims to capture the immediate actions individual banks take in response to a shock, which could very well happen simultaneously. To overcome this computationally, in each period a bank takes action while assuming that the system remains unchanged from the previous period. Once each bank has had their turn, their actions are collectively applied, altering the system, which may prompt further action by banks in the subsequent period.

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[6]They could, in principle, take other actions such as repoing out securities to get funding to pay back loans that are immediately due, but in this framework we are concerned with rare events where it is likely that the market is jittery as banks are reluctant to extend new loans to one another.

[7]"Basel Committee Revises Basel III Liquidity Coverage Ratio", Davis Polk & Wardwell, LLP, 2013

The model aims to capture the propagation, amplification, and transformation of risk throughout the financial system in a stress scenario. Due to its short time frame and available data, it focuses on the more liquid and volatile markets and relationships between banks, which motivates the balance sheet construction described above, as well as why certain banks will be more active and possibly more vulnerable in the stress test simulations. Banks heavily involved in the short-term interbank lending (repo) market and security markets will be more affected by shocks to market prices or defaults themselves, but will also be more prone to create risk for others.

Conditional on parameters and initial conditions, we have the initial state of the financial system comprising N banks, or nodes, endowed with heterogenous balance sheets and interconnections. Their exposures to other banks through the short-term interbank debt and off-balance sheet OTC derivatives markets with the corresponding securities and cash used as collateral are accounted for in their balance sheets and represented by the matrices m^s, m^{obs} and m^c, respectively.

A "period" is when every bank has a chance to react to system changes in the previous period. After all of the banks have had their turn, the effects of a default (if any) will be applied and new security prices will be calculated. At the beginning of a single instance/run of the simulation $t = 0$, a shock is applied to the system in the form of a significant negative market value change to one or more types of securities held by the banks. Initially, this will immediately depress the value of banks' holdings v_i of these securities, and thus their assets a_i, which will decrease their RWCR θ_i. On top of this, they may have posted the shocked securities as collateral against their short-term borrowings b_i^s or derivative exposures OTC_i, and may face a margin call from their creditors. If θ_i falls below the required level $\bar{\theta}$ and/or receive margin calls, they will first use their available cash c_i to satisfy immediate needs. They will then try to reduce their short-term interbank (STIB) loans d_i^s to bring up their ratio and raise cash to bolster their liquidity buffer, pay off creditors, and post cash margin.

Security prices are determined endogenously in the model, and are a function of the amount of selling in the previous period. At $t = 0$, the price of every security $s_k = 1$, but in later periods the price of each security is recalculated based on selling activity driven by banks liquidating securities to stay solvent, or by creditor banks selling collateral assets which secured their now-deliquent loan to a defaulted bank.

The endogenous price equation is based on the estimated fire sale elasticities from Begalle et al. (2013). Their numbers are based on the effect of a 200 billion dollar liquidation of an asset class, so the price equation scales their elasticity accordingly. For each security k, calculate

$$p_k^t = p_k^{t-1}(1 - \frac{\sum_{i=1}^{N} sold_k^{i,t-1}}{22}\alpha_k) \tag{8}$$

where $0 \leq sold_k^i \leq s_k^i$ is the amount of security k sold by bank i in the previous period in their effort

to stay solvent, or by creditor banks holding security k as collateral against a loan to a defaulted bank. a_k is security k's price sensitivity to selling by market participants. These new prices will directly affect the value of the collateral securing an interbank loan, and if the value of the collateral falls below the par value they will face a margin call.

A bank can face a margin call for their short-term repo borrowing by creditors or by OTC derivative counterparties. Bank i will face a margin call from a counterparty j if $m_{ji}^s - p_\kappa^t(m_{ji}^s(1 + h_\kappa)) < h_\kappa m_{ji}^s$ or if $m_{ji}^{obs} - p_\kappa^t(m_{ji}^{obs}(1 + h_\kappa)) < h_\kappa m_{ji}^{obs}$ where h_κ is the haircut for security κ, and $\kappa = m_{ji}^c$. In other words, if the value of the collateral, which starts off greater than the amount of the loan by the haircut amount, drops below the outstanding loan amount, then they will have to post margin equal to the difference between the loan and the market value of the collateral. Recall that $\kappa = m_{ji}^c$ is the type of security used as collateral in the repo transaction.

Thus the total amount owed by bank i due to margin calls is

$$mc_i = \sum_{j=1}^{N} max\{(m_{ji}^s + m_{ji}^{obs}) - p_\kappa^t(m_{ji}^s(1 + h_\kappa) + m_{ji}^{obs}(1 + h_\kappa)) \; ; \; 0\} \tag{9}$$

with $k = m_{ji}^c$ for each k.

The debtor bank will post *cash* to fulfill the margin requirement defined above and subsequent downward market price changes will be offset by further pledging of cash to their creditor. In the event that their creditor does not rollover a proportion of the repo transaction, the size of the loan will decrease accordingly. In this case, the posted cash margin will be returned to the borrower in an amount equal to the size of the loan withdrawal if the latter amount is less than the total posted cash margin. If the withdrawn amount is greater than the total posted cash margin, they will return some securities. This dynamic is important since cash and securities used as collateral remain on a bank's balance sheet, but cannot be used or sold to fulfill their various needs in a stress scenario since they are in legal custody of their creditor. The exact amounts of cash margin and securities posted and released depends on the size of loans and types of collateral used in individual transactions between banks.

To account for these details, define an entry in the STIB/repo matrix at current time τ as

$$m_{ji}^{s,\tau} = \prod_{t=0}^{\tau} \omega_j m_{ji}^s, \tag{10}$$

where ω_i^t is the amount bank i decides to withdraw depending on different obligations outlined below. (10) takes into account any withdrawals of loans from j to i in periods leading up to τ. The amount of cash c_i that is being used for margin calls to various counterparties at time τ is

$$c_i^m = max\left\{ \sum_{t=0}^{\tau} mc_i^t - \sum_{j=1}^{N}(m_{ji}^{s,0} - m_{ji}^{s,t}) \; ; \; 0 \right\} \tag{11}$$

Thus the amount of cash that is not available for use by bank i in period t is equal to the total amount of cash posted as margin since period 0, less the amount returned back to them due to a shrinkage of the loan amount.

$$c_i^{ua} = \sum_{j=1}^{N}(m_{ji}^{obs}p_\kappa^t + h_\kappa m_{ji}^{obs}) + c_i^m \tag{12}$$

In addition to the direct balance sheet losses and new obligations due to security price changes, a bank can also suffer losses when a bank who they lent to in the short and long-term interbank market defaults. This will trigger a firesale of the collateral assets, and bank i may recoup less cash from selling the collateral into a diving market than the size of the delinquent loan. This dynamic is described in detail below.

In an effort to stay solvent in response to their shocked assets and immediate obligations to creditors, banks in the previous period may have withdrawn short-term loans to raise cash and boost their RWCR. Thus it is possible that a creditor j in the STIB market will not roll over a proportion of loans to bank i, in which case bank i will have to pay them back the withdrawn amount. This is yet another obligation they must fulfill to stay solvent, and across all i's creditors is calculated as

$$b_i^{s,t^*} = \sum_{j=1}^{N} \omega_j^t m_{ji}^{s,t-1} \tag{13}$$

If they still need to boost their RWCR, raise their liquidity buffer, fufill a margin call, or return called-back funds they borrowed in the STIB, then they will have to take action. In other words, they will check if $\theta_i < \bar{\theta}$, $c_i^a < \beta(d_i + b_i^s)$, $b_i^{s^*} > 0$ or $mc_i > 0$.

They first use available cash c_i to pay back b_i^{s,t^*} and mc_i^t. If that is not enough, or $c_i^a < b_i^{s^*} + mc_i$, they will withdraw short-term interbank loans. If $\theta_i < \bar{\theta}$, they will first try to reduce short-term exposure by decreasing their repo lending as short-term loans have a higher risk-weighting than cash. The total amount to withdraw will depend on how much is needed due to each requirement.

Interbank requirement:

$$r_i^{ib} = max\{b_i^{s^*} - c_i^a \; ; \; 0\} \tag{14}$$

If $b_i^{s^*} > c_i^a$, then $c_i = c_i^{ua}$. If not, then $c_i = c_i - b_i^{s^*}$. Bank i will use their available cash to pay off short-term debt. If they do not have enough, the difference between what they owe $b_i^{s^*}$ and how much cash they have c_i is the amount to withdraw from the STIB market.

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Margin requirement:

$$r_i^m = max\{mc_i - c_i \; ; \; 0\} \tag{15}$$

Bank i will use its available cash, which was altered when trying to meet the interbank requirement, to cover their margin requirements. When posting cash margin, each piece will be paid out to each specific counterparty j depending on the size of the margin call. So each j will get $max\{m_{ji}^s - p_k^t(m_{ji}^s + h_k) - h_k m_{ji}^s \; ; \; 0\}$ from i.

Liquidity requirement:

$$r^{liq} = max\{\beta(p_i + b_i^s) - c_i \; ; \; 0\} \tag{16}$$

In other words, if bank i has paid off immediate creditors and has enough liquidity $(c_i - r_i^{ib} - r_i^m) \geq \beta(p_i + b_i^s)$, then $r^{liq} = 0$.

For needs driven by capital requirements, bank i must determine how much they can possibly restore their θ_i to reach $\bar{\theta}$ by decreasing their short-term lending d_i^s. The way they do this in the model is by considering what would happen to their θ_i if they withdrew from the STIB market in small increments until they reach $\bar{\theta}$. The resulting loan amount that will restore their RWCR is denoted by $d_i^{s^*}$. If they cannot reach $\bar{\theta}$ by withdrawing all of their loans d_i^s, they will have to move on to securities liquidation.

Capital requirement:

$$r_i^{cap} = max\{(d_i^s - max\{(r_i^{ib} + r_i^m + r_i^{sh} + r_i^{liq}) - c_i^a\} - d_i^{s^*} \; ; \; 0\}, \quad d_i^{s^*} > 0 \tag{17}$$

In other words, they first use their available cash c_i^a to cover the preceding requirements, and if they use it all up then they will withdraw the shortfall from the interbank market. r_i^{cap} takes this into account because it captures the fact that withdrawals due to the other requirements could end up restoring θ_i naturally, which will not necessitate further withdrawals for capital reasons. The amount to withdraw is thus the total loans d_i^s less the amount used up satisfying the earlier requirements, minus the ideal level $d_i^{s^*}$. If $d_i^{s^*} \leq 0$, then bank i cannot restore θ_i even by withdrawing all of their short-term loans. They will have to continue to the next stage where they will liquidate securities.

If, after accounting for the withdrawals driven by interbank, margin, and liquidity requirements, their short term lending d_i^s is low enough to restore their required RWCR to $\bar{\theta}$, then $r^{cap} = 0$. If not, they must withdraw r_i^{cap}. Thus their total requirement that will be fulfilled by withdrawing from the STIB market is

$$r_i = min\{d_i^s \; ; \; max\{r_i^{ib} + r_i^m + r_i^{liq} + r_i^{cap} - c_i^a \; ; \; 0\}\} \tag{18}$$

They will first use their available cash c_i^a to cover their requirements, and then resort to decreasing their STIB lending. If they use up their available cash and the amount to withdraw is no greater than their total amount of short-term lending, then bank i can meet their immediate needs by withdrawing funds from the STIB market. They will withdraw evenly across all counterparties $j = 1, 2, ...N$ the fraction ω_i of the total amount lent m_{ij}^s for each j where

$$\omega_i = \frac{r_i}{d_i^s}, \quad \omega_i \in [0, 1] \tag{19}$$

with $d_i^s = \Sigma_{j=1}^N m_{ij}^s$

This amount is turned into cash for bank i while the proportional amount of collateral securities become available for selling by bank j. However, if $r_i > b_i^{s^*}$, then they cannot meet their immediate obligations and fulfill their capital and liquidity ratios by solely reducing their STIB lending, so they must withdraw everything $d_i^s = \Sigma_{j=1}^N m_{ij}^s$. Set $\omega_i = 1$.

Now $d_i^s = 0$, which will have an impact on both a_i and $m^{ib}d_i^s$ in θ_i.

Liquidating securities

At this point, bank i has withdrawn d_i^s, which, assuming that none of its debtors default in the current period, they will be able to turn into cash. The next set of requirements are similar to the ones outlined above, except that bank i now assumes that they have cash to spend equal to their total short-term loans, and the amounts are computed in terms of quantity of a particular security k to be sold at the current market price whose proceeds will cover the requirement.

The decision of the order in which the security types will be sold depends on how liquid the market is and what requirement they are trying to fulfill. For example, when trying to raise cash to pay back creditors and bolster their liquidity buffer, a bank will prefer to liquidate the security k with the deepest market, i.e. with the lowest α_k. However, it may be the case that when trying to reduce their exposure to risky assets, they may choose to liquidate assets with higher risk-weightings, which will have a greater impact on their RWCR per quantity sold. The details of this process will be discussed below.

Another constraint on their security liquidation is how much of each security k is used as collateral for their short and long-term borrowings. Although these securities will show up on their balance sheets, they cannot sell them as long as they still owe money to their creditors who hold them in custody. Thus the amount of each security available to liquidate is $s_k - \Sigma_{j=1}^N m_{ji}^{sc}$ where $m_{ji}^{sc} = p_\kappa(m_{ji}^s + h_\kappa m_{ji}^s)$, h_κ and $\kappa = m_{ji}^c$.

The bank will determine how much of security k must be sold to meet the requirements driven by each of the following obligations until $s_k^a = 0$, then move on to the next security, which will be chosen to maximize the proceeds from the sale. We will see, however, that when every bank behaves this way, firesale risk can increase and reduce the efficacy of this strategy. It is also important to note that even if they run out of a security in period t, it is possible that some of that security will be released by creditors in the next period who decreased the size of their short-term loan (reverse repo) to bank i, thus freeing up more of security k to liquidate.

Define ϕ_i as the amount of available funds that bank i can apply towards fulfilling the below requirements. This amount will equal their available cash c_i^a, how much they withdrew from the STIB market $\sum_{j=1}^{N} m_{ij}^{s,t}$, and the amount from these two sources that have been allocated to the previous obligations.

$$\phi_i^{ib} = c_i^a + \sum_{j=1}^{N} m_{ij}^{s,t}$$

$$\phi_i^m = c_i^a + \sum_{j=1}^{N} m_{ij}^{s,t} - z_i^{ib}$$

$$\phi_i^{liq} = c_i^a + \sum_{j=1}^{N} m_{ij}^{s,t} - z_i^{ib} - z_i^m - z_i^{sh}$$

(1) Interbank requirement

$$z_i^{ib} = min\left\{ \frac{max\{b_i^{s^*} - (c_i^a + \phi_i^{ib}) \; ; \; 0\}}{p_k} \; ; \; s_k^{i,a} \right\} \tag{20}$$

They will have interbank needs as long as their STIB obligations are not covered by their available cash and money coming in from others after withdrawing all of their STIB lending in the previous stage. They assume at this point that everything they withdraw will be returned in full. If a debtor defaults in the current period, it is possible that the realized amount of cash they will get back will be less than the size of the loan, which they will have to make up for. This shortfall is captured in z_i^{sh} below.

Subject to the current market price of security k, the equation for z_i^{ib} will return the amount that must be sold to raise enough cash to meet their obligations towards others. If they cannot meet this obligation by selling just k, they will sell everything they can $s_k^{i,a}$ and move on to the next

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security. In this case it will be the security with the next most liquid market α_{k+1}. They will repeat this process until interbank obligations z_i^{ib} are satisfied. If they can satisfy z_i^{ib} using their cash c_i and expected inflow $\Sigma_{j=1}^N m_{ij}$ from the STIB market, then they will not have to sell any securities. If they must sell securities, cash will have been driven to zero and then technically increased by the amount of the proceeds from the liquidation $p_k^t z_i^{ib}$. However, these proceeds cannot be used since they are owed to creditors.

(2) Margin requirement

$$z_i^m = min\left\{ \frac{max\{mc_i^t - \phi_i^m \ ; \ 0\}}{p_\mu} \ ; \ s_k^{i,a} - z^{ib} \right\} \tag{21}$$

This says that if their total margin call requirement cannot be covered by their available cash or withdrawn STIB funds, then they will sell an amount of security k equal to that amount.

The next requirement captures i's obligations due to promising funds to other banks j under the assumption that their own inflows would be as expected. If their realized inflows that were pledged to counterparties are less than the amount promised in the previous period, they will have to make that up in period t.

(3) Shortfall requirement

$$shortfall_i = max\left\{ \sum_{j=1}^N \omega_j^{t-1} m_{ji}^{s,t-1} + \sum_{j=1}^N p_\kappa^{fs,t}(m_{ji}^{s,t} + h_\kappa m_{ji}^s) \ ; \ 0 \right\} \tag{22}$$

$$z_i^{sh} = min\left\{ \frac{shortfall_i}{p_k^t} \ ; \ s_k^{i,a} - (z_i^{ib} + z_i^m) \right\}, \qquad \kappa = m_{ji}^c \tag{23}$$

In the previous period, bank i withdrew an amount $\omega_i^{t-1} m_{ij}^{s,t-1}$ and assumed that they would receive it in full when deciding how much of their securities holdings to liquidate. However, if, in that same period any of their debtors defaulted, they may receive less than they expected. Since they held a collateral asset κ against loans to the defaulted counterparty, they will sell all of their holdings at the price p_κ^{t+1}. Because other banks may have lent to the defaulted bank, there will be a large amount of selling of security κ, so it is likely that $p_\kappa^{t+1} < p_\kappa^t$.

(4) Liquidity requirement

$$z_i^{liq} = min\left\{ \frac{max\{\beta(p_i + b_i^s) - \phi_i^{liq} + c_i^{ua} \ ; \ 0\}}{p_k} \ ; \ s_k^{i,a} - (z_i^{ib} + z_i^m + z_i^{sh}) \right\} \tag{24}$$

In this case we assume that even though some of their cash is posted as collateral for off-balance sheet derivative positions and as margin for interbank loans, they can consider this cash to be part of their liquidity buffer. The above equation says that if their pledged cash c_i^{ua} and available cash ϕ_i^{liq} are less than the requirement amount $\beta(p_i + b_i^s)$, then they will have to liquidate some securities. If $z_i^{liq} = s_k^{i,a} - (z_i^{ib} + z_i^m)$, then sell all of it for $p_k^t(s_\mu^{i,a} - (z_i^{ib} + z_i^m))$ so that $c_i^a = c_i^a + p_k^t(s_\mu^{i,a} - (z_i^{ib} + z_i^m))$. Then recalculate z^{liq}. Repeat until their liquidity requirement z_i^{liq} is met.

(5) Capital ratio requirement

If bank i still needs to take action to restore their RWCR, they can do so by selling securities, which account for a considerable portion of their risk-weighted assets. Decreasing that number by converting those holdings into cash, which holds a risk-weighting of zero, will increase their ratio θ_i. It is possible that in the process of liquidating securities to fulfill the preceding obligations, the ratio was improved, but if there still remains a RWCR shortfall, they will liquidate more securities. They sell security k in small increments until $\theta_i = \bar{\theta}$ or their available holdings of k runs out. They sell each portion $\Delta s_k^{i,a}$ of security k, effectively shifting the amount $\Delta s_k^{i,a} p_k^t$ from their risk-weighted assets to cash, so that $c_i = c_i^a + \Delta s_k^{i,a} p_k^t + c_i^{ua}$.

The total amount of security k that is sold in a period is

$$sold_k^{i,t} = \sum_{i=1}^{N} (z_i^{ib}(k) + z_i^m(k) + z_i^{sh}(k) + z_i^{liq}(k) + z_i^{cap}(k)) \tag{25}$$

where $z_i^{cap}(k)$ means that security k was sold satisfying that particular requirement.

Default and firesale dynamics

The dynamics described in this section take place after every bank has acted according to the foregoing process. If bank i cannot meet any of the above requirements by withdrawing from the short-term interbank market or liquidating securities, they default. Throughout the period each bank assumes that all the other previously-solvent banks will remain solvent, but after they have all had their turn to react to each other's actions from the last period according to the dynamics described above, they will have a chance to take action due to defaults. If bank i defaults, each bank j that has short-term interbank (reverse repurchase agreement) exposure to i will sell all of the collateral securities $m_{ji}^{s,t}(1 + h_k)$ and retain the total posted cash margin from i to j during the previous periods. This assumption is based on findings from Begalle et al. (2013), who found that despite the benefits of a partial and orderly liquidation of collateral assets after a default, regulations are such that individual banks have strong incentives to sell everything, exacerbating

the firesale environment as banks collectively act in a way that may end up hurting the market more. To capture this firesale risk, each j sells the collateral at a new price p_k^{t+1} that accounts for the high amount of selling so that their realized collateral liquidation proceeds is $p_k^{t+1}(m_{ji}^{s,t}(1+h_k))$ where p_k^{t+1} is calculated according to price equation. Bank j's loss λ_j^s due to their STIB exposure to a defaulted bank is equal to the difference between the loan amount and the proceeds from the sale of the collateral at the firesale price and the retained cash margin.

$$\lambda_j^s = m_{ji}^s - p_k^{t+1}(m_{ji}^{s,t}(1+h_k)) + \sum_{t=0}^{\tau} mc_{ij}^t \tag{26}$$

It is also possibile that λ_j^s becomes a slight gain for j if the price at which they sell the collateral has not fallen enough to negate the sale of slightly more collateral $(m_{ji}^{s,t}+h_k)$ than the size of the delinquent loan.

If a bank j has off-balance sheet OTC derivative exposure to the defaulted bank i, then they retain the cash collateral, which amounts to 90% of the exposure value. Their loss given default (LGD) due to these exposures is therefore 10%.

$$\lambda_j^{obs} = (0.1)m_{ji}^{obs}$$

When a bank i defaults it is not just banks that have loans and other asset-side exposures to i that are vulnerable. It is just as likely that a defaulted bank i is a creditor to another j. In that case bank j risks losing its collateral securities or cash held in custody by the defaulted counterparty, which was a major issue for many counterparties to Lehman Brothers in 2008, especially in Europe[8]. When a defaulted creditor who holds a borrower j's collateral defaults, those assets are often frozen and irretrievable by the borrower until lengthy bankruptcy processes are completed. In the time frame of the stress scenarios faced by the system in this study, bank j will consider it a loss if the value of their collateral is worth more than their assets whose purchase was funded by the borrowed funds. This loss is not direct, but puts strain on their risk-weighted capital ratios as the market value of their assets are smaller while also having less available securities to liquidate to restore it.

Another risk that a default poses to the system is through the fire sale channel. When trying to stay solvent a bank will continue to liquidate all its available-for-sale securities to try to increase their θ_i. Thus if they end up defaulting, they will have naturally liquidated all of their available securities (the rest being retained by debtors who held some of i's securities as collateral). Thus in each period, each bank evaluates losses and obligations due to a shock or activity in the previous period. If necessary, they each take action as if the system is in the same state as it was at the end of the previous period. By waiting to reflect banks' actions during period t until period $t + 1$,

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[8]Begalle et al., 2013

the dynamic of the system as if they were acting simultaneously is preserved. This looping process will continue until the system settles and no more banks have to take action due to activity in the previous period.

3.1 Channels

A primary goal of this framework is to provide insight into the evolution of systemic risk by measuring the contribution to total losses of different channels through which this risk propogates. Bank i's total loss in each period is denoted by λ_i^t, which comprises five loss components defined below. The "initial shock" channel is the direct effect of the price shock of security $k \in S$ on a bank's balance sheet, where S is the set of shocked securities. Define the indicator function $I_{[x \in \Omega]}$ on a set Ω by

$$I_{[x \in \Omega]} = \begin{cases} 1 & : \ x \in \Omega \\ 0 & : \ x \notin \Omega \end{cases} \tag{27}$$

1. Initial shock

$$\lambda_i^{is} = \sum_{k=1}^{6} (p_k^0 s_k^0 - p_k^{\xi} s_k^0) I_{[k \in S(0)]} \tag{28}$$

   where p_k^{ξ} is the shocked security price and the set $S(0) = \{k : \text{k shocked in period } 0\}$

2. Fire sales

$$\lambda_i^{fs,t} = \sum_{k=1}^{6} p_k^t s_k^t - p_k^{t+1} s_k^t \tag{29}$$

   where $p_k^{t+1} > p_k^t$ if there was significant selling of k in period t.

   The fire sale channel captures the effect of declining market prices of securities that bank i holds on their balance sheet due to selling in the previous round.

3. Funding/liquidity strain

$$\lambda_i^{b^{s^*},t} = b_i^{s^*,t}, \tag{30}$$

   recalling that $b_i^{s^*}$ is amount that bank i owes immediately due to STIB creditors hoarding liquidity. Banks' first course of action when trying to restore their RWCRs θ_i is to withdraw funds from the repo/STIB market. If a bank borrowed from i, they will have to come up with the cash, and their balance sheet will shrink by that amount. This is a likely point where the risk in the system changes, since this funding strain may force a bank to liquidate securities, which can then cause margin calls and fire sale losses. Cycles like these continue until the system reaches an equilibrium or they all default.

4. Post-default fire sale

$$\lambda_i^{pdfs,t} = \sum_{j=1}^{N}(m_{ij}^{s,t} - p_{\kappa}^{t+1}(m_{ij}^{s,t}(1+h_{\kappa})))I_{[j \in D(t)]} \tag{31}$$

where $\kappa = m_{ij}^{c}$ is the type of security used as collateral, and $D(t)$ is the set of banks who defaulted in period t.

This channel captures fire sale losses that are specifically due to widespread selling of securities used as collateral for loans to a defaulted bank. Because these loans are overcollateralized, however, it is possible that a creditor may profit from this situation if the fire sale loss does not exceed the extra profit from the haircut.

5. OBS OTC derivative losses

$$\lambda_i^{obs,t} = \sum_{j=1}^{N}(0.1) \cdot m_{ij}^{ib} \cdot OTC_i \cdot I_{[j \in D(t)]} \tag{32}$$

Since all OBS OTC exposures are collateralized 90% by cash, the loss due to this channel will be 10% of the exposure size.

Thus the total loss for bank i in a period is

$$\lambda_i^t = \lambda_i^{is} + \lambda_i^{fs,t} + \lambda_i^{b^{s^*},t} + \lambda_i^{pdfs,t} + \lambda_i^{obs,t} \tag{33}$$

and total loss over the course of a trial is $\sum_{t=1}^{T}\lambda_i^t$ where T is the number of periods it takes for the system to reach equilibrium. To supplement bank risk-management tools and regulatory stress tests, this model is concerned with the feedback effects of an intial shock, which are nontrivial in the scenarios considered below. We can separate these effects by subtracting the initial shock λ_i^{is} from the total loss $\sum_{t=1}^{T}\lambda_i^t$.

3.2 Simulation dynamics and data collection

Monte Carlo simulation is used with this model to estimate the effect of a shock on the financial system. We have balance sheet data and banks' involvement in the interbank markets, but there is not publically available data on the myriad bilateral exposures between them. This interbank network is very important for how risk moves throughout the system, so the model simulates over a range of possible network structures.

Bank balance sheet compositions and the initial shock are the same for each trial, while the base interbank exposure network and collateral type matrix will be randomly generated. The

banks will behave subject to the framework described above, and the trial will terminate when the system reaches its steady state. The variation between the trials is due to the varying strengths of interbank linkages and the type of securities used as collateral for the repo (STIB) market. The base interbank exposure matrix determines both the proportion of each banks repo lending and borrowing with one another, as well as the off-balance sheet over-the-counter derivative contracts. Contagion due to the initial shock will depend on the strength of these interbank linkages and the types of securities used for collateral. A bank who has posted a large amount of their MBS holdings as collateral, for example, will be vulnerable to a shock to MBS prices, as they will face margin calls, and in the case that they need to liquidate securities to stay solvent, they will have less MBS available to sell since some of it will be held in custody by their creditor.

The model collects detailed information about each bank's balance sheet, constraints, obligations to other banks, and their actions in each period of a single trial. Depending on the goal of the stress test, the model collects data in different ways. After an entire Monte Carlo simulation (1000 runs) the model will summarize the data and produce average losses due to each channel with confidence bands, as well as the probability of default under the stress scenario. To gain more insight into the evolution of the different risks, the model consolidates data from a single trial to demonstrate how the different channels change over time as banks take action to tend to their balance sheets.

4 Data and its mapping to the model

4.1 Balance Sheet Data Mapping

Bank balance sheet data are based on the quarterly regulatory filing, FR Y-9C, by bank holding companies to the Federal Reserve Board. The FR Y-9C contains consolidated balance sheet information, as well as more detailed information on security holdings and off-balance sheet exposures. It also contains information on the banks risk-weighted assets and regulatory capital. Because an important constraint that the agents face in the model that guides much of their behavior is the maintenance of their risk-weighted capital ratio θ_i, only institutions that report this information in the FR Y-9C are considered. The top 10 banks by asset size are used in the model, so the institutions that are excluded due to this are AIG and GE Capital, which rank 7th and 8th, respectively.

The focus of the model is on the short-term propagation of risk due to common exposures to securities, OTC derivative contracts, and short-term interbank collateralized lending and borrowing. The balance sheet data in the FR Y-9C is consolidated in order to isolate the pertinent items for the model. The asset-side of each bank's balance sheet consists of cash c_i, short-term loans, or

Model	FR Y-9C
UST	U.S. Treasury Securities
USGA	U.S. Gov't Agencies, State and Political Subdivisions Obligations
MBS	MBS (Residential, Commercial, Agency, Non-Agency, CMO)
ABS	ABS, Structured financial products
Other debt securities	Other debt securities
Equity	Equities, investments in mutual funds

Table 1: Balance sheet consolidation from FR Y-9C

Asset	Risk-weight	Elasticity
UST	0	2%
USGA	0.2	2%
MBS	0.5	2%
ABS	1	5%
Other debt	1	5%
Equities	1	8%
Interbank lending/repo	0.5	-
OBS OTC derivative exposures	1	-
Other Assets	0.5	-

Table 2: Security risk-weightings and fire sale elasticities

repurchase agreements d_i^s, securities s_k, and other o_i^a. Total cash and repo numbers are used from the FR Y-9C, while s_k is a consolidation of their "Trading Assets" and "Securities." The security types in the FR Y-9C are mapped to six groups of securities as in Table 1. The actual balance sheet numbers are scaled down by a factor of 1.09698E-07, which was chosen so that Goldman Sachs' assets would be \$100. GS was chosen because their assets were roughly the median size in the ten bank sample.

4.2 Exogenous Parameters

This model relies on a number of parameters that remain fixed throughout the simulation exercises, which include asset risk-weightings, security market price elasticities, and haircuts used for repo collateral. The balance sheet data used in the model matches the aggregate numbers from the FR Y-9C, but there is significant consolidation of different line items into broader categories. Based on Basel III regulations[9], each banks assets are weighted according to Table 2.1.

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[9]See www.federalreserve.gov/bankinforeg/basel/BaselCommitteeDocuments.htm

Because of the relatively high-level data on risk-weighted assets in the FR Y-9C, as well as further consolidation of asset types in the model, the "other" risk-weighted assets θ_i^o is chosen so that each bank's RWCR θ_i matches the actual ratio θ^{act} reported in the filing[10].

When a bank borrows from the short-term interbank market, they enter into repurchase agreements, where they pledge securities as collateral in exchange for cash. The haircut is the amount of extra security the borrower will pledge as protection to the creditor against adverse market movements, expressed as a percentage of the loan amount. The Federal Reserve provides monthly data on collateral types, volume, and haircuts in the tri-party repo market. Haircuts range from 2% to 8%, but it is assumed that each security in the model has a haircut of the median 5%[11].

Another important set of parameters are the price elasticities of the types of securities with respect to selling in the market. The degree to which widespread selling of a particular asset depresses prices drives the fire sale channel of contagion and depends on these parameters. Begalle et. al (2013) study the risk of fire sales in the tri-party repo market in the U.S., and in examining the structure, statistics, and regulations surrounding the market, they estimate the risk of fire sales in different asset classes used as collateral. Based on the size and trading volume of each market, they use a VaR analysis to arrive at potential shortfalls a bank would face if they tried to liquidate a type of security used as collateral during a fire sale situation. The elasticities α_k are based on the median percentage shortfalls.

5 Results

5.1 Stress testing

Banks hold on their balance sheets a wide variety of financial instruments whose prices are sensitive in different ways to market forces. Understanding as much as possible about their balance sheet faced with market uncertainty and carefully guiding their profit-seeking behavior to reduce the chance of significant losses is the difficult goal of risk management.

VaR analysis is widely used by banks to give them an idea about the different risks their balance sheets face over some period with some degree of statistical confidence. These analyses are often based on historical data, and in "normal" market conditions perform adequately. In "abnormal" circumstances, however, these tools are very poor measures of risk, which was made particularly apparent in the most recent financial crisis, and even arguably a contributing factor. Understanding the risk of rare events like these is even more important since that is when the consequences of underestimation are the most dire.

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[10]$\theta^o = \frac{a_i - l_i}{\theta^{act}} (w^{ib} d_i^s + \Sigma_{k=1}^6 w_k s_k^i p_k^t + w^{obs} OTC_i)$

[11]sensitivity analysis to come

Stress testing is an important supplement to traditional tools, especially during periods of relative quiet as the chance of complacency of decision makers is highest and statistical models' memories of stressed times are the faintest. New regulations and the recent financial crisis have made stress tests an integral part of continual risk-monitoring by regulators and banks as they are both very interested in how their balance sheet will fare during times of market stress. Stress tests vary widely in complexity and and how they choose input risk scenarios. Under the Dodd-Frank Act, the Federal Reserve requires banks to run very complex stress tests that involve dozens of input variables over long time periods. However, instead of a bank focusing on just their own portfolios and how they expect the value to change due to scenarios, albeit very comprehensive ones, the framework outlined above can inform this approach by considering the important feedback effects during a stress scenario. Even if a bank knows how their portfolio will change due to some shock, it is just as important to have some expectation of how they will react and how collective action of other players will subsequently affect their balance sheet.

5.2 Stress testing within the model

In addition to providing insight into market feedback effects, the model can attribute losses to different transmission mechanisms. Understanding the different ways that a seemingly simple shock can eventually affect a balance sheet gives a more holistic picture of a firms different risks and guide preventative action. For example, due to an initial downward shock to an asset a bank is holding, they may take some action that will likely prompt other market participants to take action. The interconnectedness of the financial system all but ensures the propagation of these shocks throughout the system. The risk posed to a bank due to an asset shock can manifest into liquidity hoarding, which could prompt others to sell securities to raise liquidity, which could then return to the "trigger" bank in the form of further asset price declines or margin calls. The stress test below is an example of how a shock to a few securities can turn into different types of risks as it moves around the system, with the overall damage being greater than the initial shock. The model aims to put a number on these knock-on effects and to provide insight into the process of risk propagation and transformation.

5.3 Simulation results - Bank perspective

Large market value declines in one asset class are often accompanied by market declines in other, especially related, markets. The stress scenario considered is a 30% decline of MBS prices, equities, and other debt securities, simulated 1000 times. Each trial begins with the same shock, but the interbank exposure network and the types of securities used as collateral for repurchase agreements are randomly generated, driving the variation of the outcomes from each multi-period run. This

Measure	5th	Mean	95th
Total Loss	-11.029	-9.705	-8.650
Initial Loss	-5.299	-5.299	-5.299
Excess Loss	-5.73	-4.406	-3.351
Fire sale	-0.811	-0.657	-0.594
Post-default fire sale	-0.528	-0.059	0
OBS OTC	0	-0.007	-0.042
Funding	-4.993	-3.690	-2.701
Margin Call	0.098	0.658	1.255
Number of defaults		0.2%	

Table 3: BoA stress test simulation results (30% downward shock to MBS, Equities, and Other Debt)

example is from the perspective of Bank of America (BoA), the second-largest bank by assets in U.S., and the results are summarized in Table 3.

The initial loss due to the pure market value shock to their security holdings is 5.3%, but their average total loss is 9.705%. The additional 0.66% loss over the pure effect was due to further market value declines of securities they held due to fire sales, and 3.7% was due to liquidity/funding problems. Due to their short-term creditors not rolling over repo transactions, they were forced, on average, to come up with cash equal to 3.7% of their balance sheet. This forced reduction of their liabilities reduces the size of their balance sheet, but likely causes BoA to withdraw funds from the short-term interbank market and sell securities themselves, contributing to the fire sale asset prices and causing problems for other banks who depend on their short-term lending. In this particular scenario, the post-default fire sale and off-balance sheet exposure channels did not have a very significant role. The small contribution of the OBS OTC exposure is likely due to the fact that BoA was not heavily exposed to banks that defaulted, and to the fact that those exposures are collateralized 90% by cash in the model. The post-default fire sale is also small due to the over-collateralization of short-term loans, and can even lead to gains as BoA can sell an amount of securities greater than (haircut $h = 0.05$) the size of the loan. In this scenario the timing of these post-default collateral sales are such that the price of the collateral had not fallen enough to cause shortfalls when the time came to liquidate.

To give a more detailed picture of the dynamics of this process, we can focus on a single trial of the simulation and watch the different channels interact and the balance sheet composition
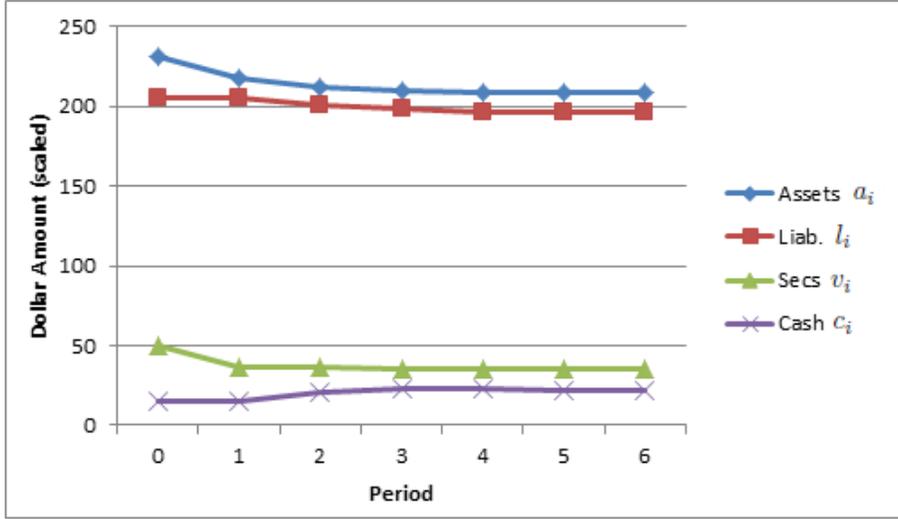
Figure 1: Balance sheet composition over time during a single trial

change over each period. Figure 1 shows a relatively high-level view of BoAs balance sheet over time, plotting total assets, liabilities, total securities holdings, and cash levels. Figure 2 plots their obligations and constraints in each period, including the size of their margin calls, how much they must come up with to pay back short-term creditors (funding strain), how much cash they have posted as margin, and how much of their securities portfolio is being used as collateral in their repurchase agreements. Observe in Figure 1 how their assets fall due to the initial shock and subsequent fire sales in period 1. Their cash level rises over this period as they withdraw cash from the repo market (do not roll over their reverse repo transactions) and sell securities.

Figure 2 gives a more detailed view of the forces driving these aggregate changes. The asset shock triggered margin calls, causing more of their cash to be tied up as margin. At the same time, other banks withdrew short-term loans from BoA (Funding Strain), which released many of their securities from custody making them available for sale. At the same time, however, this reduced liquidity likely caused problems elsewhere as they are forced to convert their assets into cash to repay their immediate short-term creditors. If they have enough cash on hand that is not tied up as margin, they can use that, otherwise they will withdraw from the STIB market themselves or liquidate securities.

Figure 3 breaks down BoA's losses in each period by channel. Most of their loss in the first period is due to the initial shock, but the total loss is a bit higher do to fire sales by BoA and other banks as they tried to stay solvent. The endogenous prices of each security in each period are plotted in Figure 4, which mirrors the losses that BoA incurs in each period due to market value
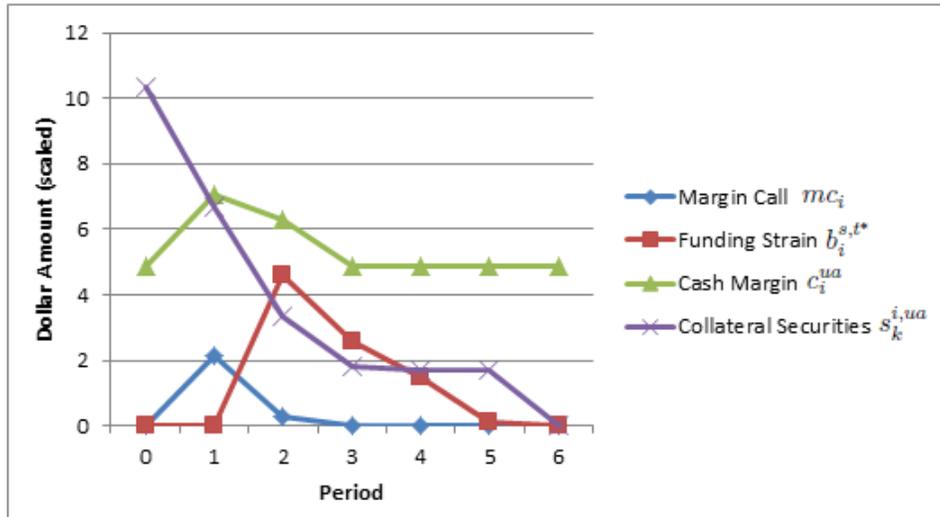
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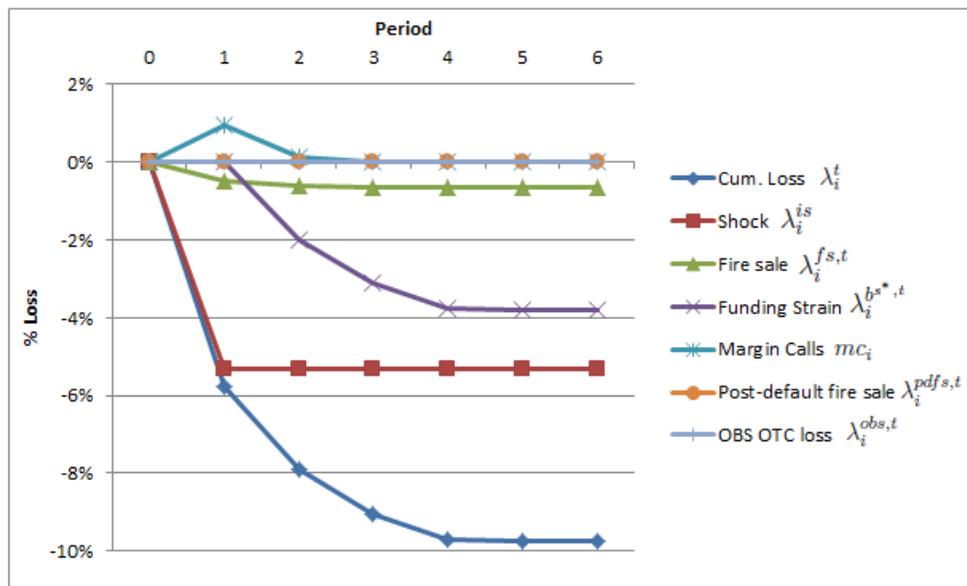Figure 2: Constraints and obligations over time during a single trial

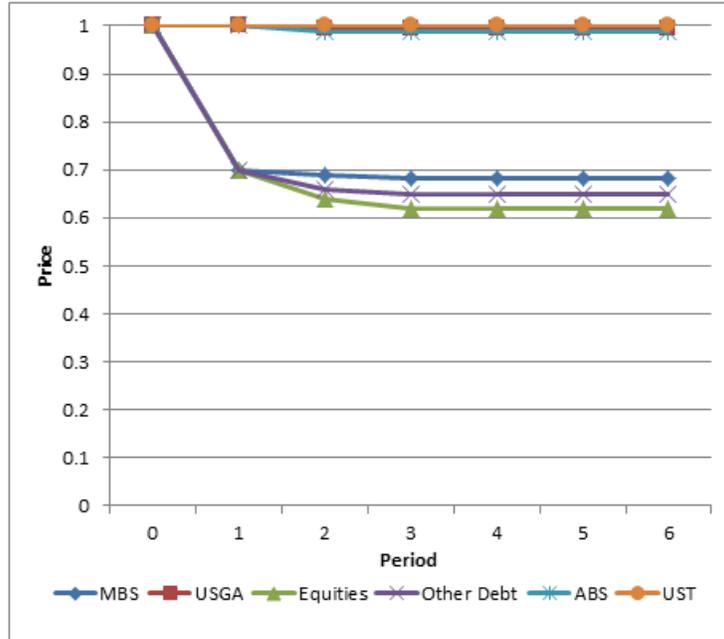Figure 3: Losses, broken down by channels over time

29

Figure 4: Security prices in each period

declines of their security holdings. Also observe that much of their total loss is driven by other banks withdrawing short-term liquidity from the market in later periods.

Figure 5 demonstrates how banks make decisions with respect to one of their primary constraints of maintaining a risk-weighted capital ratio θ_i of at least 8%. θ_i is plotted over the course of the trial for Bank of America and Goldman Sachs. Differences in their size, strength of connections with other banks, and balance sheet compositions drive the different actions they take to satisfy the constraint. GS holds a large amount of trading assets on their balance sheet, including large amounts of the shocked securities (MBS 5.3% of assets, Equity 10%, Other debt 6.6%, BoA: 11%, 3%, 3.5%). This explains the more pronounced effect of the shock on GSs θ_i initially, which causes them to immediately hoard liquidity, withdrawing all credit from the interbank repo market, which restored their ratio temporarily. Due to the actions taken by other banks in the system, however, their θ_i fell back down to 4% at the end of period 1. They proceeded to sell available securities in period 2 until it was restored again, which was enough to cushion further feedback losses such as fire sales and short-term loan repayments for the rest of the periods.

Bank of America, on the other hand, stayed above the minimum RWCR requirement $\bar{\theta}$ after the initial shock, but actions taken by others in the first period drove it down below the threshold. They were able to restore it by withdrawing 45.3% of their credit from the STIB market (unwind
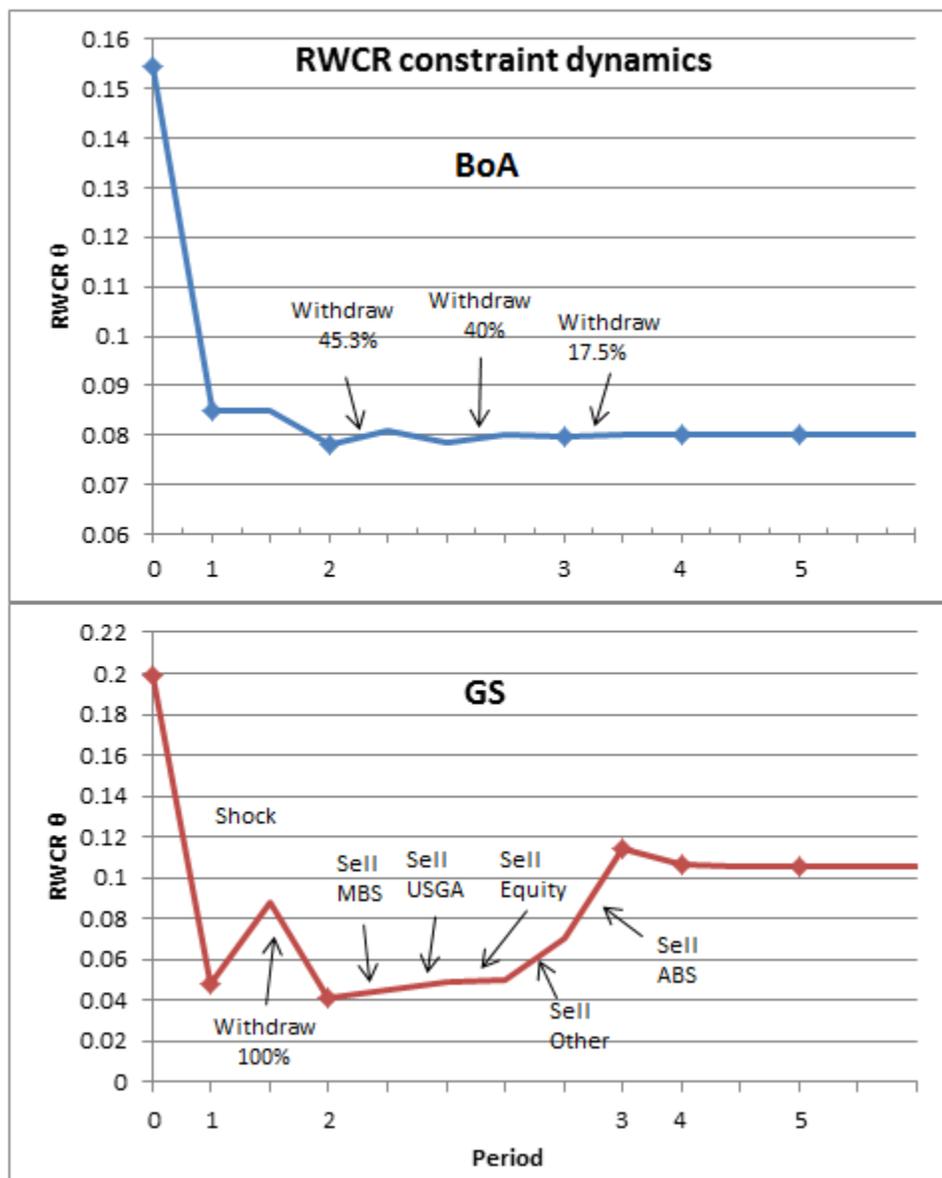
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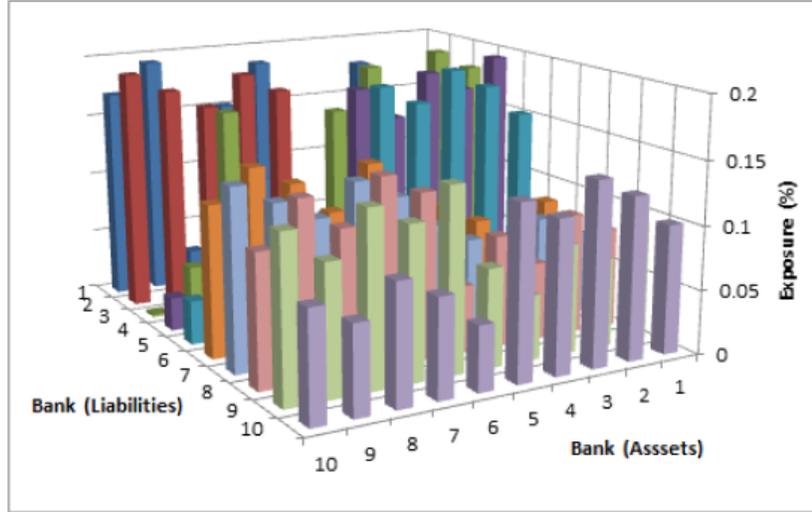Figure 5: Individual bank behavior to maintain the minimum RWCR over time

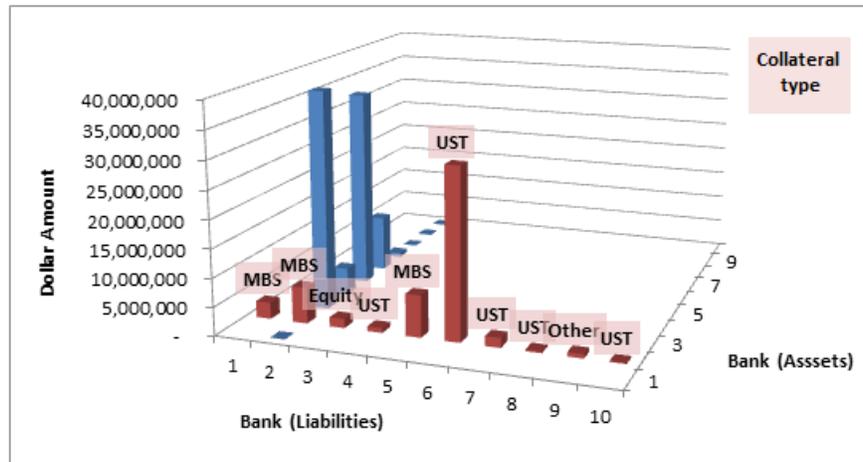Figure 6: Percentage exposure each bank has to one another (asset and liability-side)

Figure 7: BoA's actual ($) exposures ($m^s$) with collateral security type

reverse repo transactions). Feedback effects brought θ_{BoA} down again in the next period, which they were able to restore with further withdrawals. This process continued until the fourth period where no action was necessary subject to this particular constraint. It is still possible that they faced other requirements such as their buffer size $\beta(p_i + d_i^s)$, margin calls mc_i, and funding strains $b_i^{s^*}$.

Figures 6 and 7 show the structure and collateral composition of the short-term interbank network for this particular trial of the simulation. Each bar in figure 6 is the percentage of bank is total short-term loans that is to bank j. Notice that any bilateral exposure is capped at 20% of a banks total exposure, as well as the high degree of connectivity, but also plenty of asymmetry. Figure 7 is cross section of this interbank matrix, where the labeled bars are BoAs liabilities (repos) and the corresponding collateral security type. This information can be useful for a bank running a stress test like this, since it can help explain variation between the different risk channels. For example, if much of their collateral consisted of equities, then a large market value decline will hit them harder than if their posted collateral was more diversified, and with knowledge of the rest of the collateral network, they can take preemptive action to reduce risk.

VaR Analysis

The limitations of Value-at-Risk models for risk management are well-known, a principle weakness of which is their failure to perform in times of market stress. Besides their reliance on historical data, a primary reason for this is that they do not take into account the abnormal situations such as liquidity shortages when assessing their expected losses. Despite its shortcomings, VaR analysis remains a ubiquitous risk management tool, and the framework in this paper can help inform these models. By integrating the agent-based network model into a VaR framework, the model can take into account the feedback effects from the system. The choice of underlying distribution driving the VaR analysis is not the focus of this paper, but given a distribution, the model can include the knock-on effects that may arise from certain random shocks.

In addition to accounting for the feedback effects in the system, the framework described here can allow for a more detailed understanding of the VaR results. On top of the usual expected loss with a degree of statistical confidence, the model can decompose the expectation into the different risk channels. The following example demonstrates how this model can be integrated into a VaR framework in a very simple case. We assume that equity price returns follow a normal distribution with mean 0 and standard deviation $\sigma = 0.1819^{12}$. The other securities k in the model are assigned arbitrary correlation coefficients ρ_k relative to equity prices, where $\rho_{MBS} = 0.2$, $\rho_{USGA} = 0.2$, $\rho_{O.D.} = 0.5$, $\rho_{ABS} = 0.6$, and $\rho_{UST} = 0.0$. The bank used in this example is Goldman Sachs.

The distribution is drawn from 100 times, and for each draw, the simulation over 1000 interbank

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[12]Based on S&P 500 prices from July 7, 2008 to July 7, 2009

Total Loss	Initial Shock	Fire sales	PDFS	OBS OTC Loss	Funding Strain	Margin Calls
-8.84428	-4.875	-1.5	0	0	2.459722	0.495189

Figure 8: VaR results for Goldman Sachs
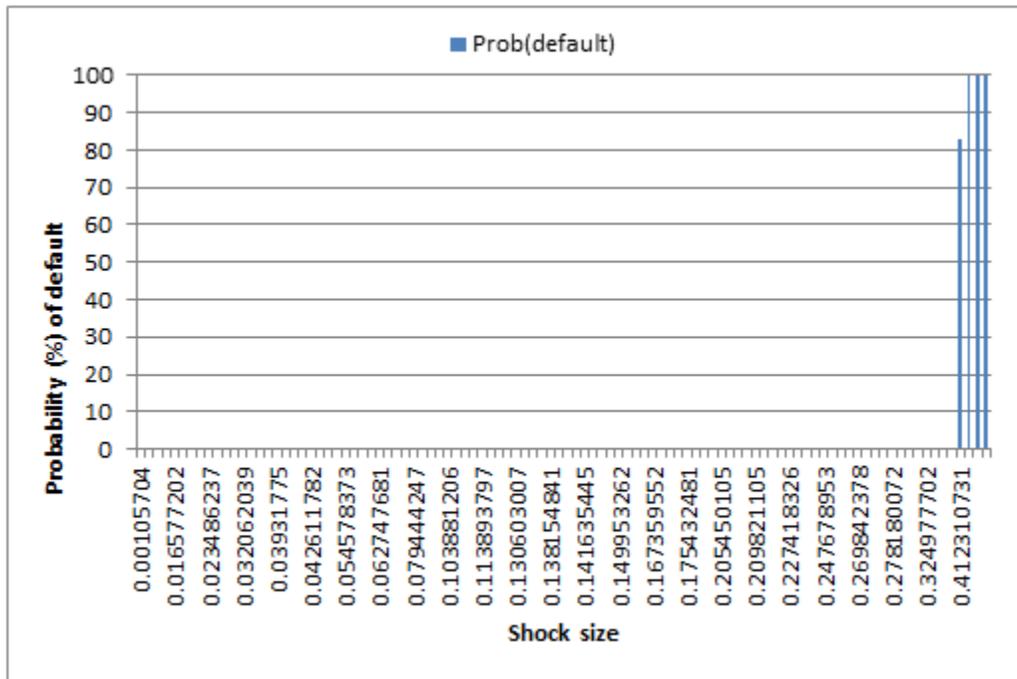
Figure 9: VaR results - probability of default (MS)

Measure	5th	Mean	95th
Total Loss	-10.440	-9.026	-7.214
Initial Loss	-5.347	-5.347	-5.347
Excess Loss	-5.093	-3.679	-1.867
Fire sale	-0.738	-0.498	-0.411
Post-default fire sale	-0.219	-0.025	0
OBS OTC	0	-0.004	-0.022
Funding	-1.081	-2.985	-4.344
Margin Call	0.092	0.609	1.134

Table 4: Regulator stress test results for aggregate risk in entire system

networks is run. For each shock the average loss due to each channel and the number of defaults are recorded. The 99th worst outcome, broken down by channel, is then reported as the potential losses with 99% confidene. The results are shown in Figure 6. From this particular, simplified scenario, the model picks up feedback effects that account for 46% of the total losses. Again we see that funding liquidity is an important driver of losses, as well as fire sales. Figure 7 shows the probability of default for Morgan Stanley given the same distribution of equity returns and correlation coefficients. Observe that at certain draws the number of defaults increases sharply, from 0 to 80, in this case.

5.4 Simulation results - Regulator's perspective

This framework can also be used by regulators for stress testing, which offers the advantage of taking into account the multiple channels through which financial institutions are connected, and also providing insight into the relative contribution of these different channels as well as individual institutions' contributions to this risk factors. A regulator might be interested in estimating how the financial system would fare under a severe market downturn like the one considered above. Using the same shock of 30% to MBS, Equities, and Other Debt prices, we analyze the effect of this on the system as a whole. Table 4 summarizes the simulation and Table 5 shows the number of times each bank defaulted out of the 1000 trials. Observe that in aggregate it is often the case that the feedback effects from the interaction of different agents over the course of a trial amplify the ultimate effect of the initial shock.

In addition to understanding how a shock propagates and amplifies as it spreads throughout the

Bank	Defaults
JPM	216
BoA	2
Citi	0
WF	0
GS	130
MS	260
BNYM	0
USB	947
PNC	0
CapOne	820
Total	*2375*

Table 5: Regulator simulation results - number of defaults out of 1000

system, a regulator might want to know the roles that the market participants play in this transmission mechanism. Figure 8 plots each banks relative contribution to the different channels from which losses manifest as a percentage of total assets in the system. Figure 9 displays contributions relative to each banks own balance sheet. As one would expect, the larger banks in the system contribute the most to the overall loss to the system in the simulation, with JPM, Citi, GS, BoA, and MS suffering the bulk of the aggregate losses and funding/liquidity strains. However, controlling for balance sheet size in the bottom figure we see that MS and GS have a disproportionate effect on total losses, exceeding those of JPM, BoA, Citi, and WF. Wells Fargo, in particular, despite its size, fares well in the simulation. The concentration of losses on the pure investment banks, GS and MS, make sense however, since the stress scenarios tested and channels examined by the model focus on the liquid, public financial markets and short-term interbank relationships, in which the investment banks participate heavily.

6 Conclusion

As the global financial system grows in size, innovates, and becomes increasingly interconnected, the need for a more sophisticated understanding of the ever-changing risks will become increasingly important. The model presented here uses an incomplete sample of the U.S. financial system and leaves out some important financial institutions. Like any theoretical model, its applicability is only as strong as the assumptions and parameters that guide the dynamics of the system. Combining the natural platform of networks in finance with an agent-based approach has promise, and even the simple results demonstrated here can inform existing stress testing methodologies, which, despite
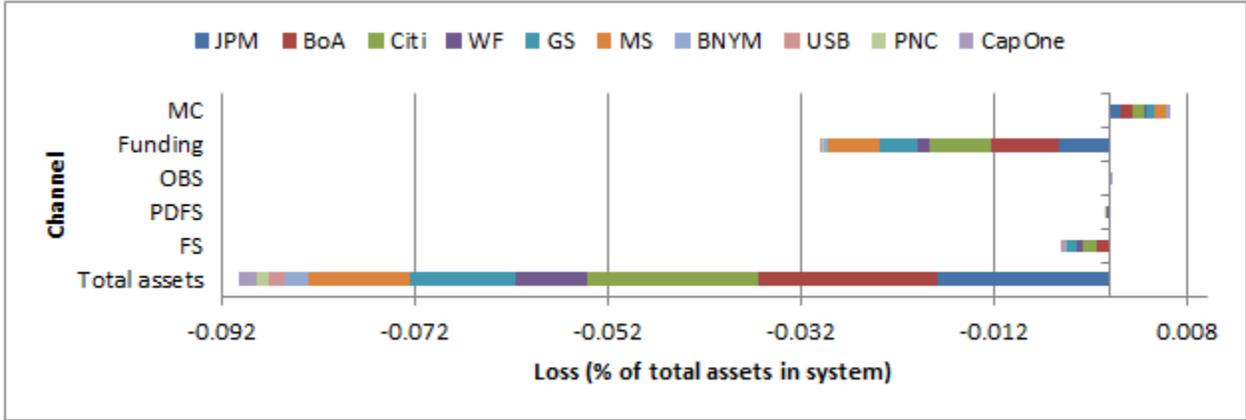
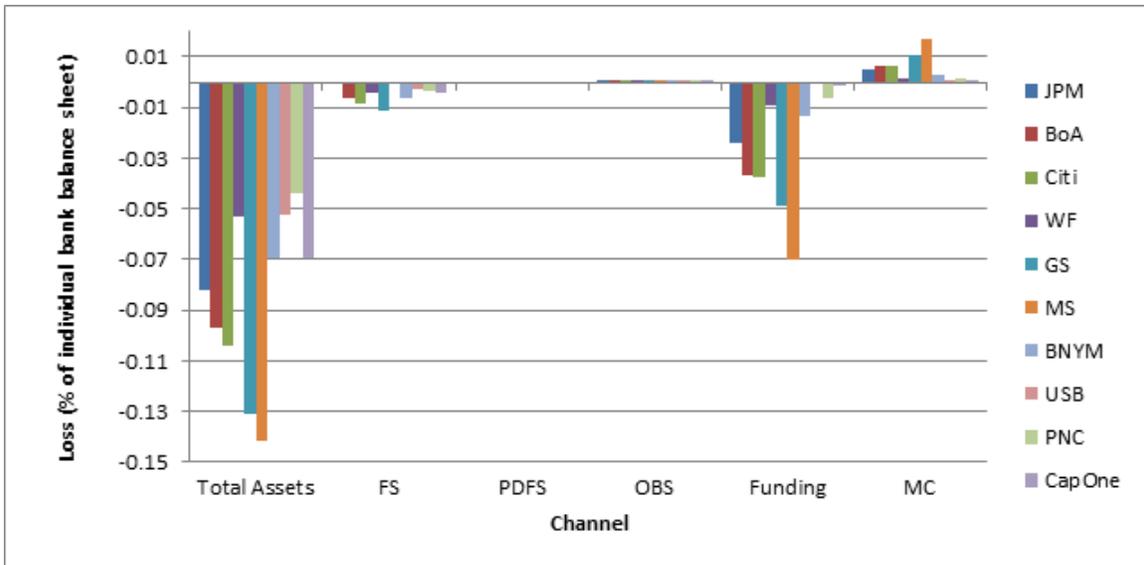Figure 10: Bank contributions to risk channels (% total system assets)

Figure 11: Bank contributions to risk channels (% individual bank) balance sheets

their comprehensiveness, are also prone to neglect the equally important feedback effects during a stress scenario. From a bank's perspective, having an idea of how they and others will react in response to a potential shock they may face allows them to prepare for risks that are not directly apparent. By tracking these feedback effects through time a bank can take preventative action to reduce their overall risk. Similarly, if regulators have a more granular understanding of risk propogation in the financial system they can make more informed and targeted regulations.

The stress tests carried out here showed that the interbank lending market is a major source of systemic risk in the U.S. banking system, and the collateral underlying these transactions links this risk to potential fire sales and margin calls. Observing the dynamics of a single trial of the simulation revealed that these effects often do not arise until later periods, after the intial shock. It was also shown that this model can be integrated into a VaR analysis framework for risk management, potentially ameliorating one of VaR's limitations of failing to account for feedback effects due to large shocks found in the "tails" of a chosen distribution.

An advantage of this framework is its flexibility, since a wide array of stress test scenarios can be used. Only a few possibilities were discussed above, but another interesting scenario that regulators might be interested in is a liquidity crunch scenario where the interbank debt market dries up suddenly, which was shown to be a primary driver of systemic risk. Testing the effect of credit risk transfer products on systemic risk is also a very pertinent topic for regulators. When individual institutions trade these securities, the risk in the system can become concentrated, creating conditions that leave the system vulnerable to crises. In addition to being easily extentable by using more information on market structure and bank behavior, a framework like this is well-suited to to be integrated into existing approaches used by banks and regulators.

References

[1] Allen, Franklin, and Douglas Gale. "Financial Contagion." *Journal of Political Economy* 108.1 (2000): 1-33. Print.

[2] Begalle, Brian, Antoine Martin, James McAndrews, and Susan McLaughlin. "The Risk of Fire Sales in the Tri-Party Repo Market." *Federal Reserve Bank of New York Staff Reports* (2013).

[3] Bookstaber, Richard. "Using Agent-Based Models for Analyzing Threats to Financial Stability." *Office of Financial Research, U.S. Department of the Treasury* (2012).

[4] Chan-Lau, Jorge A. "Balance Sheet Network Analysis of Too-Connected-to-Fail Risk in Global and Domestic Banking Systems." *IMF Working Paper* (2010).

[5] Cont, Rama, Amal Moussa, and Edson Santos. "Network Structure and Systemic Risk in Banking Systems." Working Paper (2012).

[6] Duarte, Fernando, and Thomas M. Eisenbach. "Fire-Sale Spillovers and Systemic Risk." *Federal Reserve Bank of New York* (2013).

[7] Eisenberg, Larry, and Thomas H. Noe. "Systemic Risk in Financial Systems." *Management Science* 47.2 (2001): 236-49. Print.

[8] Elsinger, Helmut, Alfred Lehar, and Martin Summer. "Risk Assessment for Banking Systems." *Management Science* 52.9 (2006): 1301-314. Print.

[9] Montagna, Mattia, and Christoffer Kok. "Multi-layered Interbank Model for Assessing Systemic Risk." *European Central Bank* (2013).

[10] Nier, Erlend, Jing Yang, Tanju Yorulmazer, and Amadeo Alentorn. "Network Models and Financial Stability." *Journal of Economic Dynamics and Control* 31.6 (2007): 2033-060. Print.

[11] Upper, Christian. "Using counterfactual simulations to assess the danger of contagion in interbank markets," (2007) *BIS Working Papers 234, Bank for International Settlements.*

[12] Upper, Christian, Worms, Andreas. "Simulation methods to assess the danger of contagion in interbank markets." *European Economic Review* 48.4 (2011):827-849.

Appendix A

Interbank Network Construction

A financial system by nature is highly connected as it fosters liquidity in markets and the creation and allocation of different risks. These connections, however, are a primary source of risk to the system becuase problems for one institution can quickly become problems for others because of this interdependence. Despite its importance, however, there is no detailed data on these interbank linkages in U.S., so this structure will be randomly generated in the model. A base interbank network will be used to determine the size of linkages between counterparites in the repo market and the off-balance sheet over-the-counter derivative market (OBS OTC), and is denoted by m^{ib}. This interbank network can be represented by the matrix whose entries m_{ij}^{ib} correspond to an exposure of bank i to bank j. In the case of the repo market, m_{ij}^{ib} would correspond to a collateralized short-term loan from i to j. Similarly, bank i would also be interested in their obligation towards j, m_{ji}^{ib}. The value of each entry in the interbank matrix corresponds to a percentage of i's total exposure to j. This imposes the important constraint that each row and column sum to one. In other words, a bank's proportional lending and borrowing to and from all of the other banks cannot exceed 100%. Thus we must have a matrix where $\sum_{i=1}^{N} m_{ij}^{ib} = 1$ and $\sum_{j=1}^{N} m_{ji}^{ib} = 1$. This is not a trivial feature to require a matrix to have, but a branch of recreational mathematics called "magic squares" provides much of the machinery needed to produce matrices conforming to our constraint. A magic square is a square matrix whose rows and columns all add up to the same number whose value depends on the size of the square and the range of numbers chosen to populate the matrix. To produce an interbank matrix of proportional exposures, a random magic square is generated, and each entry is divided by the row sum to yield a percentage.

Banks cannot lend to themselves, however, so it is imposed on m^{ib} that $m_{ii}^{ib} = 0$ for all i. It is assumed that the proportional exposure that is removed from a row and column by forcing a diagonal entry to be zero represents a bank's exposure to the "rest of the market," not explicitly treated in the model. Although the ten banks considered in this model account for the majority of activity in the markets of interest, this assumption acknowedges that the included banks are not entirely dependent on just the other 9 banks.

6.1 Mapping the interbank network to bank balance sheets

The interbank network is used to construct the short-term interbank lending (STIB), or repo market denoted by m^{s}, where m_{ij}^{s} represents a collateralized loan from bank i to j. Whereas m^{ib} is defined in terms of percentage exposure, m^{s} is defined in terms of actual dollar amounts. Due to the varying balance sheet sizes and compositions of the different banks, it is possible that a large bank

i's dollar exposure to a smaller bank j $m_{ij}^s = m_{ij}^{ib} d_i^s$ could exceed even the total amount that the smaller bank has borrowed from the interbank market. To account for this, the construction of m^s from m^{ib} chooses each m_{ij}^s so that

$$
m_{ij}^s = \begin{cases} d_i^s & : \ d_i^s m_{ij}^{ib} \leq b_j^s m_{ij}^{ib} \\ b_j^s & : \ d_i^s m_{ij}^{ib} > b_j^s m^{ib} ij \end{cases} \tag{34}
$$

In other words, if bank i's proposed dollar amount of loans to bank j exceeds the amount that j is supposed to be borrowing from i, then the dollar amount is set to the smaller amount, which in this case is j's borrowing. The remaining loans $d_i^s m_{ij}^{ib} - b_j^s m_{ij}^{ib}$ from i that are not applied to j are allocated to the outside market.

A consequence of this set-up will be that, as we would expect, actions taken by large banks will have greater dollar effects on smaller counterparties despite sharing the same percentage exposure. When m^s is constructed according to equation (34), it will likely be the case that a small bank such as Capital One's borrowing and lending will be fully allocated to a larger bank such as JPM, whereas JPM's proposed percentage lending to Capital One would be much larger than Capital One's, and be subsequently alloted to the outside market. When JPM defaults, for example, the percentage exposure of Capital One to JPM will be much greater than the effect of Capital One's default on JPM.

For each run of the simulation, a different interbank network is generated. There are three different networks in the banking system that are derived from the random interbank matrix. The STIB market matrix m^s has a corresponding collateral matrix m^c where each m_{ij}^c is the type of security used as collateral for the loan m_{ij}^s. Banks are also connected through off-balance sheet over-the-counter derivative transactions. The FR Y-9C form reports each bank's net "current credit exposure" to counterparties in these positions and the type of collateral they are holding against that exposure. Current credit exposure, or replacement cost, is the market/fair value of the derivative contract if it is positive, and zero otherwise, measuring the cost for bank i to replace the position if their counterparty were to default. This captures counterparty risk in the large off-balance sheet OTC derivative market, which, due to the inherent leverage in derivative transactions and their existence "off-balance sheet" is an important channel through which contagion can spread that is much larger than balance sheet sizes in the system would suggest. The matrix m^{obs} represents this network where each m_{ij}^{obs} is the off-balance sheet credit exposure bank i has to j. Since the magic squares generation of the random, base interbank matrix yielded a complete network between the banks in the model and an "outside entity," each m_{ij}^{ib} is a percentage of bank i's total exposure allocated to j. This network is static throughout each run of the simulation.

The repo network is derived from this matrix by distributing each bank's repo lending and borrowing to other banks according to the underlying m^{ib}. Unlike in other interbank network models, each entry of the matrix m^s will depend on which bank is using it, and if they are evaluating asset-side exposures or liability-side exposures. That is, bank i's repo agreement with j in dollar terms is $m^s_{ij} = m^{ib}_{ij} d^s_i$, where d^s_i is i's total lending in the repo/STIB market. Similarly, if i is interested in their obligation to bank j, they compute $m^s_{ji} = m^{ib}_{ji} b^s_i$.

For the off-balance sheet exposure network, bank i is interested in their current credit exposure to another bank j, which is similarly calculated using the underlying random interbank matrix so that $m^{obs}_i j = m^{ib}_{ij} \cdot OTC_i$.

6.2 Generating the collateral matrix

Each entry $m^c_{ij} \in \{1, 2, 3, 4, 5, 6\}$ in the matrix m^c of collateral asset types corresponds to the type of security held against a loan from bank i to j. This matrix is derived probabilistically using tri-party repo data from the Federal Reserve. The data break down the types of securities used as collateral in tri-party repo transactions as a percentage of the total. According to the data-mapping described in section 4, the percentages ψ_k from the FRBNY are interpreted as the probability that a given security type will be used as collateral for a STIB loan. Thus each entry of the collateral matrix m^c_{ij} is a random variable with probability mass function

$$f(k) = \psi_k \tag{35}$$

where $k \in \{1, ...6\}$ is the security type and ψ_k is the share of total Tri-Party repo lending that was collateralized by security k in Q4 2013.

Another important network in the financial system is the network of common exposures to securities. As in Montagne and Kok (2013), consider a matrix m^v where each entry m^v_{ij} represents the correlation between bank i and j's asset holdings. In other words, it is the degree to which a shock to the market price of an asset held by two different banks will affect the overall system. These exposures are implicit in the model as banks' shared exposure to assets is a consequence of their balance sheet compositions, this shows that a connection between two financial institutions does not have to be an explicit transaction, and is an important dimension to the overall structure and dynamics of the system.

Appendix B

Interbank matrix generation algorithm and statistics

The interbank exposure matrix is generated used techniques from *www.perfectmagicsquares.com*, which provides information on algorithms for creating magic squares of different sizes. The 10x10 matrices used in the model are generated using four random 5x5 perfect squares and combining them in a way that preserves the property that all row and columns sums are equal. The numbers in each cell range from 0 to 99 and the "magic sum" is 495. Thus an entry of m^{ib} of 24 corresponds to a $24/495 = 4.8\%$ exposure from i to j.

There are 275,305,224 different 5x5 magic squares, including rotations and mirrorings. The number of possible 10x10 matrices is therefore greater than this, providing an ample amount of possible networks used in the Monte Carlo simulation. The matrices are generated randomly in each trial by producing a random 5x5 matrix, which involves choosing two random rows that form the bases for the eventual magic square. The 5x5 square is then rotated or mirrored randomly. This random 5x5 is then turned into a 10x10 matrix according to the techniques described in the website. A final random element is introduced by randomly rotating or mirroring the 10x10 matrix.
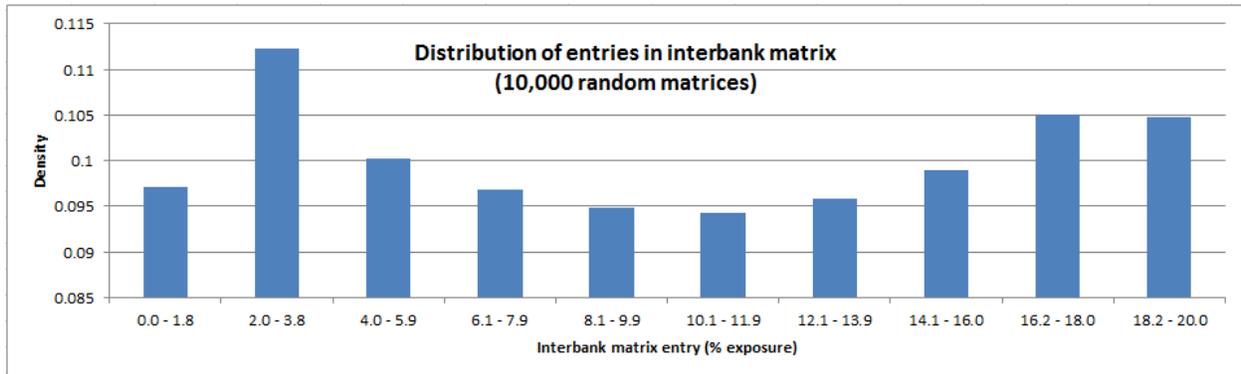
Average minimum	Average Maximum
1.78%	18.02%

Each entry of the 10x10 matrix represents a percentage exposure to another bank. The histogram above is based on 10,000 random 10x10 matrices, categorized by percentage ranges. There is fairly even distribution of exposures, with some bias towards the upper and lower ranges. The average maximum exposure of any one back towards another was 18.02% and the average minimum exposure was 1.78%.

Appendix C

Normalized Balance Sheets used in the simulations

	JPM	BoA	Citi	WF	GS	MS	BNYM	USB	PNC	CapOne
Assets	264.9	230.9	206.3	167.5	100	91.35	41.06	39.93	35.17	32.61
Cash	39.34	15.08	21.82	22.73	8.729	8.743	16.07	0.929	1.77	0.706
Repo	39.39	22.07	28.19	3.122	37.13	27.19	0.996	0.009	0.170	0.000
Securities	65.01	49.71	55.20	32.43	29.01	29.44	9.65	4.568	5.669	4.798
Other	121.2	144.1	101.1	109.2	25.13	25.98	14.35	34.42	27.55	27.11
Liabilities	241.7	203.4	183.7	148.8	91.37	83.77	36.83	35.35	30.33	28.04
Deposits	141.3	123.0	106.2	118.4	7.755	12.08	28.65	28.75	24.27	22.47
Rev. Repo	19.78	21.71	22.23	3.863	20.12	19.58	1.03	0.226	0.459	0.099
Other	80.72	60.64	55.22	26.47	63.48	52.12	7.152	35.35	5.604	5.467
OBS	7.49	5.431	4.955	0.251	11.48	4.634	0.714	0.001	0.096	0.053
RWCR	14.36	15.44	16.65	15.43	19.87	16.94	16.97	13.20	15.78	14.73